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## The Design of Relationship Model between (IRAN) Economic Markets Return and Capital Market Return Exploiting Comonotonicity in Probability Theory

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### Abstract

This paper investigates the design of an efficient model so as to anticipate the basic economic market rate of returns. To do so, accepting the relationships, interactions and effectiveness of these markets and exploiting Comonotonic Functions under Probability Function Framework as well as using weekly data for ten years' period of time(2008-2017) in Iran's economy we design optimum model and test its capability and estimation power. The results illustrate the efficiency of the achieved model. Furthermore, taking the practical nature of this paper into account, we come up with optimum lag of time and the period of time required to achieve equilibrium in any market and the entire economy as a prototype in the frame of Stock Exchange.

**Keywords:** Behavioral Finance, Economic equilibriums, Comonotonic, Rate of return, Systematic risk.

## Introduction

The velocity of changes, competitive atmosphere in the economy, existence of markets, diverse investment opportunities, interconnectedness and relationships among them render investors and decision-makers in the economy to need specialty and tools for precise recognition and empowerment in prediction and sensible decision making at the right time.

One of the most considerable issues which has invariably attracted investors to select an investment portfolio is a type of investment which provides them with access to profitable opportunities, a maximum of returned and a minimum of risk having resource constraints. Consequently, investors and decision-makers have been attempting to identify the best investment opportunities and risks, returns on identified opportunities having systematic and nonsystematic risks in mind and also a select optimal combination of investments as well as intelligent management based upon the constant flow of information of the investment on the portfolio.

Identification of investment opportunities plus investigation and selecting them in multitudinous markets seem to be one of the necessities and essential measures which investors heed. Variety of returns and corresponding risks of diverse markets behavioral communication, effectiveness and interaction among them bold the significance of gaining knowledge over their behavior and relationship among them a great deal more than the past. Over the last two decades, the concepts of management of investment and finance have been developing swiftly among investors and decision-makers in I.R.Iran. Attention to two concepts of risk and return as the management principle of investment and finance has been put to trial and error through research and executive measures. Mathematical paradigms and diverse tools have been exploited and developed in this regard.

On the other hand, particular attention has been paid to the concept of an efficient market as "Theory of Efficient Market". Numerous studies and tests have been conducted to estimate the efficacy of Capital market. The theorists who formed such theories believed we could appraise the real value and come up with the correct pricing by analysis of economic and financial variables in each efficient market. Based upon this theory no investors can gain returns systematically within long terms more than returns corresponding to the accepted risk. In such markets, the price of assets reflex the related information and changes in prices lack a particular pattern and are unpredictable. Opposing this view, analyses and econometric studies and inspection of asset prices in different markets over the last three decades a paradigmatic expansion and

development evolved called Financial-Behavioral whereby regarding evidence and exceptions psychological phenomena play a crucial part in determining investors' behavior and consequently the price and returns on assets as well as investment opportunities in markets (Barbris & Taller, 2003). This branch of behaviorism and the phenomenon of psychological influences in the determination of asset prices in different markets within the financial-behavioral framework have been taken into account in the Iranian economy over the recent decade. Nonetheless, the focus of such studies and actions whether nationwide or the ones conducted overseas has always been directed toward Capital market or specified Stock exchange. The key act in this study is the issue that when a decision is to be made to invest, the entire opportunities in different markets plus the relationships impacted by systematic effects of the whole economy are taken into consideration. Thus, an efficient model is required to provide us with the interrelation of investment returns dynamically irrespective of restrictive hypotheses. This model is also supposed to provide us with efficient projections of alterations in Capital market concerning continual and dynamic changes in competitive markets.

Given the mentioned subjects and the raised questions in this paper, we intend to detect and introduce markets enjoying investment opportunities in the economy along with Capital market via return relationship modeling among these markets and to put forward a usable framework and a pattern to investors and decision-makers. To do so, computational means and infrastructure are of great importance. In the present paper with the assistance of Comotonic Functions in Probability theory, we investigate an efficient model which recognizes return relationships among markets and can forecast the change of returns on each Capital market as an effective factor in decision-makers' orientation. In Probability theory, Comotonicity refers to completely positive independence of components of an accidental vector. It indicates that Comotonic Function portrays such relationships as incremental functions of the unit accidental variable. This concept could be widely applied in the management of financial risk and actuary science (Puccetti, Giovanni Scarsini, Marco (2010).

### **Theoretical Principles and Research Background**

Given the new aspects of the subject and the objective of this study i.e. the design of relationship model between economic markets return and capital market return exploiting comonotonicity in probability theory to select optimal investment portfolio restricted by conditions in Iranian economy whether locally or overseas, the corresponding records of this study are classified into two levels:

First level: Undertaken studies about the investigation into relationships between diverse markets in an economic market with a financial-behavioral approach

Second level: Undertaken studies over the last two decades concern the application of Comotonic Function in financial management and economy related to the inspection of relationships in various basic markets, Capital market with acceptance of identifiable relationships, interactions among these markets within the financial-behavioral framework. One of these comprehensive studies was made by David Nawrocki and Fred Viole in 2014 within financial-behavioral framework and theories of financial management. Nawrocki and Viole looked into the financial-behavioral item in the theory of the financial market, the theory of optimality, portfolio theory and the required statistics. In this study they reject the hypotheses ruling modern financial management based on market returns. They illustrate that in reality not only the economy of markets maintain meaningful relationships and interactions, but within a framework of theories of optimality following any change of behavior in every market in an entire body of the economy these interactions drive them towards a stable equilibrium. There is also another study in the same domain conducted by Livio Stracca which looks into financial-behavioral status, asset pricing and the development course of this domain. Livio Stracca displays that the core of asset pricing along with the impact of changes to prices in markets and consequently changes in returns move toward a new equilibrium.

Accepting the relationships among markets, it appears truly remarkable to know the appropriate methods and tools used in probability theory, mathematical tools for modeling and ability to predict market returns considering their interrelationships.

In connection with relationships among the basic markets of economy, along with the studies mentioned here, another survey was carried out by Zeid Ftiti, Ibrahim Fatnassi, Aviral Kumar Tiwari in 2016. They investigated into novel finance, financial-behavioral concept and behavioral reflection of deals assessing both oil market and gold market together. They applied the Wavelet method to compare and contrast short-term and long-term behavior.

With respect to basic markets of economy exploiting financial-behavioral literature, a study conducted by Saskia ter Ellen and Remco C.J. Zwinkels captures meticulous attention for its modeling method of such relationships. Adopting a financial-behavioral approach to heterogeneous modeling, they inspected the price of oil in connection with other markets. In this field at this level, several studies have been worked on about Iranian economy over the

past recent years. One of these studies was set out to examine the correlation among prices of assets in markets of the economic base using Wavelet method by Nima Pazuki, Shahpur Mohamadi and Vahid Mahmudi in 2012. Another study was undertaken by Akbar Esfahani titled "Analysis of behavioral reflection in the Iranian Capital market within financial-behavioral literature".

Given the local and international studies undertaken in line with the objective of this paper into the identification of returns Comonotonicity of diverse markets and designing a relationship model, we carried out some inspection. With respect to the second-level studies into Comonotonic Function and its applications in financial and economic management, Zuo Quan Xu conducted a survey in 2014. In his survey, he developed a mathematical model upon quantiles hypothesizing positive relationships between pricing core of asset and financial-behavioral item. He demonstrated that the output of every decision is based on the concept of accidental vectors depending upon a model of sharing risks which is interpretable. To expand probability theories and Comonotonic Function in financial-behavioral field Giovanni Puccetti and Marco Scarsini studied and produced a multi-variable model of Comonotonicity. Both these studies enjoy models upon probability theories for extending a relationship model between pricing core of diverse markets using the concept of Comonotonicity (positive and negative) so as to produce a model which is able to anticipate market returns.

Given the elaboration previously on this paper, we attempt to dynamically identify the relationship among returns on different markets to estimate and come up with an informed decision about investments and to lead macro decisions toward a stable equilibrium in the economy whilst there is not a comprehensive model to describe the relationship between Capital market returns and other competitive markets to forecast expected investment return. With intent to have maximum noise suppression and to promote precision in designing the target model utilizing Comonotonic Function within probability theories, we have endeavored to eliminate restrictive hypotheses regarding its extensive application in financial risk study and actuary science. In fact, we have based the principle of this study on the acceptance of the mighty relationships among competitive markets in an economy as a foundation to expand the target model and the necessity of a comprehensive model to aid decision-makers and investors. To follow it up, we will benefit from the experience and background of past studies to achieve a target model. Apart from designing the model, we will also use data from identified basic markets over the last decade to gauge the power of the designed model.

## Research Questions

In this paper we raise research questions instead of research hypotheses:

Given the significance of necessity of developing a relationship model between competitive markets and Capital market in the economy, effective features in the efficiency of the model and the application of probability theory and in particular Comonotonic Function the main question raised in this paper is:

Considering the returns on Capital market and those of other Basic markets, which model can be designed to predict investors' desirable rate of return?

Asking this question and to come up with an appropriate reply as well as designing a practical some minor questions are posed:

What models could be used to predict investors' desirable rate of return? How is the relationship between the returns on the Capital market and those of other Basic markets? What models could be used to establish a relationship between the returns on the Capital market and those on the basic market? Which identified two-part models are valid in line with the condition of the study course? How is the two-part model which leads us to select an investment portfolio? Given the significance and application of this research, the outcomes of the research benefit all management activists, economic managers, researchers and those who are interested in investment and Capital market particularly Ministry of economy, Central bank and investing Companies. The research geographically covers basic economic markets and Capital market in IRIran. Time-wise it covers the data and information existing over the last decade ending in 2016. The subject of the research conforms to the design and description of the relationship model of returns between basic economic markets and Capital market index using Comonotonic Function in probability theories to select optimum investment portfolio.

## Research Methodology

Since research methodology is considered as the base of each research and as every scientific research requires a specific scientific methodology so that their results obtain confirmation, we will describe the present research methodology in this section. To do so we will be dealing with the methodology and the studied model. This research is made up of two statistical communities. The first community concerns basic economic markets for a period of 10 years ending in 2016. The second community concerns investment markets including

Stock exchange and over the counter market whose component of the return indicators are represented by cash return indices and prices. Using the conducted research and the thematic literature, basic economic markets below have been designated as pillars of the first statistical community:

Money market – Currency market – Housing market – Gold market

As for Money market, the interest rate index has been selected as the return indicator variable of this market. Given that annual interest rates are introduced to Money market approved by Council of Money and Credit, a one-year rate for each year within a period of 10 years ending in 2016 is considered as the basis for the interest rate. As it is necessary to match time periods used in the research, the mentioned rate has been generalized taking note of 52 weeks a year. As for the Currency market, three key foreign currencies i.e. US Dollar, Euro and Pound are regarded as target variables, however; there are plenty of foreign currencies available. In reference to these variables, weekly periods are taken into account for this research.

As for the Housing market, housing price index announced by the central bank in Iran has been selected as a basis for calculating returns. For this index, since statistical figures are not officially announced regularly especially in some periods, we have employed matching method for these periods to retrieve data on a weekly basis. As for the returns on the Gold market, the statistical rate is obtained from the Central bank of Iran so that we work out weekly returns.

As for Capital market, both Stock exchange and OTC market are considered as the major pillars of Capital market as it is based upon financial Bonds. To estimate the return rates on either market, cash index and the price over the research time period are obtained and the return rates are assessed weekly. Selecting research methodology depends upon objectives, research subject and its executive facilities. The decision on inspection and doing a survey is made when the subject, objectives and the range of the survey are all clear. In other words, the purpose of research methodology for the researcher is to determine which method helps them to work out research questions more quickly and more precisely (Ezati, 1997). Scientific/practical research; which is also called a solution, is to solve a problem directly. Practical research applies to a certain area and cannot be extended to other domains. Such research fulfills humans' needs using cognitive foundation and fundamental input. There is a wide variety of methodology available for practical research such as historical, descriptive, correlative, causative and empirical to name as important ones (Hafeznia 1998). 'Development research' is a type which

serves the purpose of innovation and development in processes and production tools or enhancing the quality of products. Development research is rarely found in the economy or social science but is common to engineering; nonetheless, they are observable in social science (Ezati 1997). Considering the purpose of the present research which is based upon “the design of relationship model between economic market return and capital market return exploiting comonotonicity in probability theory” to select optimal investment portfolio, in terms of purpose, this research is regarded as development-practical. Moreover, in terms of methodology, it is descriptive- surveying exploiting mathematical tools and modeling.

The framework and the sequence of the research could be explained as follows:

Initially, based on comparative analysis and thematic literature we identify some competitive basic economic markets which offer investment opportunities along with Capital market. Next, we select the return index in compliance with the chosen mathematical model and we collect the required data. Using comparative analysis, thematic literature, appropriate tools, Comonotonic Function and taking probable restrictions into account we design the model. Having the outputs from the previous stages of the model, hypotheses and the Comonotonic Function, we estimate the desired returns on the investment due to differences of returns on the Capital market and those on other basic economic markets. Then, we administer Reliability test and Relevancy test on the model using Comonotonic Function. At this stage, we render an optimal portfolio including investment opportunities in various markets.

Since in modeling, using comonotonicity between dependent and independent variables regarding symbiosis and integration could lead to false regression we avoid this matter in the present study. Thus, via Comonotonic Function and econometrics, we refine and purify the data.

### **Data Analysis**

Here we will elaborate on the practical model to analyze data taking notice of the mentioned principles in the research history, thematic literature and research methodology. Specifications of data will be discussed in terms of reliability and unreliability at the outset of this section. If the data is not adequately reliable, the results of the models cannot have conceptual nor statistical credit. Determination of effects among variables is another means to determine the causality so as to forecast the variables in the model. Granger



causality is a method to clarify the causality among the variables of the model. The acquired relationships to estimate the structure of VAR (Vertical Auto-Regressive) fail to describe the effects among the variables because relationships result from reduced form and this relationship demonstrates the Multiplier and we cannot exactly recognize the change of which variable results in the produced coefficient. As a result, Granger causality is employed to come up with effects among variables. What matters in systematic models is the effects of the dynamic system resulting from the change in variables of the model. To obtain such results, impulse response action functions (action and reaction functions) under probability theory toward Comonotonic Functions in the model are used. Determination of proper lag in the model plays a crucial part in systematic models. As in systematic models we consider further freedom than single-equation models to estimate the degree, determination of lag of endogenous variable in the model are quite significant as exogenous variables. This research adopts AIC and BIC criteria (Akaike Information Criterion, Bayesian Information Criterion (Schwarz Criterion) to determine the proper lag in this model.

**Statistical Interpretation of Data**

Here descriptive and broadly-applicable parameters from statistics are provided. To report the general status of the research data the diagram indicates average, variance, standard deviation, minimum and maximum as statistical figures below.

Table 1. descriptive and broadly-applicable parameters

Variable	Sign	No. of observation	Average	Standard Deviation	Minimum	Maximum
Dollar	UED	520	0.004	0.01	-0.02	0.09
Euro	EUR	520	0.004	0.01	-0.02	0.08
Housing	HOS	520	0.008	0.01	-0.03	0.13
Pound	GBP	520	0.015	0.03	-0.07	0.18
Stock Exchange	BR1	520	0.003	0.004	0.002	0.004
OTM	BR2	520	0.004	0.03	-0.08	0.16
Interest rate	INT	520	0.06	820	0.07	0.024
Gold	Gold	520	0.004	0.03	0.12	0.18

Given the statistical distribution of target variables, we will be dealing with the preparation of research variables to elaborate on the research model and designing of the simulated scenario. Primarily, reliability of variables will be investigated to ensure there is no false regression, later; optimal lag of the model will be determined. The model of regression will then be corrected. Next

section involves Action and Reaction Functions. Analysis of Variance is to be encountered in the final section.

### Reliability Test

Since data analysis depends upon the reliability of data time-wise, we primarily run Dickey-Fuller test to gauge reliability/ unreliability of data. We could also run the Phillips Perron test which seems rigorous. As the subject data is not extensive, the Dickey-Fuller test' validity is assured. The test results are illustrated below:

Table 2. Reliability Test of Data

Variable	Degree of Probability on Surface	Result	Degree of Probability	Result
Dollar	0.3514	unreliable	0.000	reliable
Euro	.05665	unreliable	0.000	reliable
Housing	.6444	unreliable	0.0002	reliable
Pound	0.0003	reliable	-----	-----
Stock Exchange	0.0022	reliable	-----	-----
OTM	0.000	reliable	-----	-----
Interest rate	0.2891	unreliable	0.000	reliable
Gold	0.000	reliable	-----	-----

The unit root test result indicates that variables of Pound, Stock Exchange, OTM and Gold are reliable on the surface. Other variables should be subject to a reliability test. Reliability test results in the first-time changes show that they became reliable following the first-time changes and modeling can be developed based upon the variables.

### Determining the Number of Optimal Lags

We are trying to produce a model in accordance with factual data without being affected by classic theory, complying with comonotonicity and exploiting Comonotonic Functions under probability theory, therefore, we ought to use statistical figures to come up with the optimal lag of the model. These statistics are calculated as follows:

A: (Akaike Information Criterion) AIC

$$AIC = \log \frac{1}{n} \sum_{i=1}^N e_i^2 + \frac{2K}{N} \quad (1)$$

B: (Schwartz-Bayesian Information Criterion) BIC

$$BIC = \log \frac{1}{n} \sum_{i=1}^N e_i^2 + \frac{K}{N} \log N \tag{2}$$

The model is developed based upon amassed data and the lag which partly includes smaller AIC and BIC is preferred and optimal. The smallness of statistics in this model reveals that addition of one new variable has made up the cost due to a decrease in returns (as a result of an increase in variables) by decreasing as much as SSR. BIC statistics is a more detailed index than AIC statistics to observe over 100 figures. It is noteworthy that the use of these two principles necessitates that economic theory provides no guidance on selecting the optimal model. Optimal lag is determined in compliance with various criteria. Naturally, several criteria have been reported to ascertain the optimal lag in this model. As per the table below, the first lag seems to be the most appropriate one for this model which is reasonably investigated considering the market conditions at a specific time. The sign (\*) represents optimal lag. Thus, the important three criteria for the first lag are regarded as the optimal lag of the model.

Table 3. Determination of Optimal Lag

HQIC	SBIC	AIC	EPE	DF	LR	LL	Lag
-5.5157	-5.5157	-5.5721	.000013	0.00	4	157.235	No lag
-5.5955*	-5.7330*	-5.8495*	.00001	0.894	4	179.137	First lag
5.4570	-5.7256	-5.895	9.5e-06	0.06	4	174.113	Second lag

**Clarification Of Model**

We will estimate our model while taking notice of specifications of Comonotonic Functions based on continuity of vectors and effectiveness of data vectors from each market on specified markets using one of the most applicable probability Functions under Comonotonic Functions.

Autoregressive vector model is a statistical model which reveals the linear dependence among series of times. Autoregressive vector model is the generalization of Autoregressive model to model the dependence among more than one time series. In Autoregressive vector model, the future of a time series and the past and other time series are estimated. From mathematical point of view, if we suppose, then ‘i’ indicates the time series in time of ‘t’, and  $X_i(t) \in R^{d \times 1}$  indicates all the time series in time of ‘t’. Autoregressive vector model illustrates the dependence between ‘X (t)’ as follows:

$$x(t) = \sum A(t)x(t - l) + \epsilon(t) \tag{3}$$

As explained in the previous section, since Action and Reaction is likely to exist among variables, Autoregressive vector technique is used and we assume that every variable could be considered as response variables in the model, in other words, VAR shall be spelled out as follows:

$$Y_t = b_{10} - b_{12}z_t + c_{11}y_{t-1} + c_{12}z_{t-1} + c_{13}x_{t-i} + \varepsilon_{yt} \quad (4)$$

$$Z_t = b_{20} - b_{22}y_t + c_{12}y_{t-1} + c_{22}z_{t-1} + c_{23}x_{t-i} + \varepsilon_{zt} \quad (5)$$

Whereby,  $Y_t$  and  $Z_t$  are response variables in the model above and in correspondence with the model above according to the definition in the present model are included as *endogenous* variables in the model.  $X_{t-i}$  and any relationship in every exogenous variable are included in the model. It is concluded that:

$$\begin{aligned} \text{GBP}_t = & b_{10} - b_{12}\text{GBP}_{t-i} + c_{11}\text{INT}_{t-i} + \\ & c_{12}\text{HOST}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR2}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{17}\text{GOLD}_{t-i} + \varepsilon_{\text{GBP}} \end{aligned} \quad (6)$$

$$\begin{aligned} \text{USD}_t = & b_{10} - b_{12}\text{USD}_{t-i} + c_{11}\text{INT}_{t-i} + \\ & c_{12}\text{HOST}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR2}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{17}\text{GOLD}_{t-i} + \varepsilon_{\text{USD}} \end{aligned} \quad (7)$$

$$\begin{aligned} \text{INT}_t = & b_{10} - b_{12}\text{INT}_{t-i} + c_{11}\text{USD}_{t-i} + \\ & c_{12}\text{HOST}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR2}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{17}\text{GBP}_{t-i} + \varepsilon_{\text{INT},t} \end{aligned} \quad (8)$$

$$\begin{aligned} \text{HOST}_t = & b_{10} - b_{12}\text{HOST}_{t-i} + c_{11}\text{USD}_{t-i} + \\ & c_{12}\text{INT}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR2}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{17}\text{GBP}_{t-i} + \varepsilon_{\text{HOS},t} \end{aligned} \quad (9)$$

$$\begin{aligned} \text{EUR}_t = & b_{10} - b_{12}\text{EUR}_{t-i} + c_{11}\text{USD}_{t-i} + \\ & c_{12}\text{INT}_{t-i} + c_{13}\text{BR1}_{t-i} + c_{14}\text{BR2}_{t-i} + \\ & c_{15}\text{GOLD}_{t-i} + c_{16}\text{GBP}_{t-i} + c_{17}\text{HOST}_{t-i} + \varepsilon_{\text{EUR},t} \end{aligned} \quad (10)$$

$$\begin{aligned} \text{BR1}_t = & b_{10} - b_{12}\text{BR1}_{t-i} + c_{11}\text{USD}_{t-i} + \\ & c_{12}\text{INT}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR2}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{16}\text{GBP}_{t-i} + c_{17}\text{HOST}_{t-i} + \varepsilon_{\text{BR1},t} \end{aligned} \quad (11)$$

$$\begin{aligned} \text{BR2}_t = & b_{10} - b_{12}\text{BR2}_{t-i} + c_{11}\text{USD}_{t-i} + \\ & c_{12}\text{INT}_{t-i} + c_{13}\text{EUR}_{t-i} + c_{14}\text{BR1}_{t-i} + \\ & c_{15}\text{BR1}_{t-i} + c_{16}\text{GOLD}_{t-i} + c_{16}\text{GBP}_{t-i} + c_{17}\text{HOST}_{t-i} + \varepsilon_{\text{BR2},t} \end{aligned} \quad (12)$$

$$\text{GOLD}_t = b_{10} - b_{12}\text{GOLD}_{t-i} + c_{11}\text{USD}_{t-i} +$$

$$c_{12}INT_{t-i} + c_{13}EUR_{t-i} + c_{14}BR1_{t-i} + c_{15}BR1_{t-i} + c_{16}BR2_{t-i} + c_{16}GBP_{t-i} + c_{17}HOS_{t-i} + \varepsilon_{GOLD,t} \quad (13)$$

‘ε ...t’ Include meaningless and trivial relationships in the model and it is called White noise. To facilitate the computation, we try to write the all equations as a whole and then the final relationships will be achieved through a reduced form. The pre-multiplication of the reversed matrix coefficients of endogenous variables in two sides of the equation result in the equation above. In the equation if we consider the coefficients two by two in blocks ,in other words if the matrix of coefficients which include five rows and columns are divided into a matrix which include two rows and columns, then the result would appear as:

$$\begin{bmatrix} b_{10} & 1 \end{bmatrix}^{-1} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

In which, the final relationship indicates one unit of change in lag of each endogenous variable on the endogenous variable. We ought to be aware of the fact that existence of a meaningful relationship in the model above does not represent existence of causality effect as this model is only the response to the question that if there is a meaningful relationship among included variables in the model. This introduction paves the way to description of the results from the estimation of the equation.

### Estimation of Structural Model

The structural model above has been estimated via Maximum likelihood estimation using the data related to mid-2006 and mid-20016 on a weekly basis. The results of the estimation reveal that the estimated algorithm has converged at 218.61 and the estimated parameters have also been reported. Moreover, the results demonstrate that  $F = 18.76$  and due to the value of  $F$ , the table approves 5% of the meaningful model.

$$\begin{aligned} D(USD) &= 0.183405835607 * D(USD(-1)) - 0.153170880957 * \\ D(HOS(-1)) &+ 0.00333390712563 * GBP(-1) - 0.012695949368 \\ *GOLD(-1) &- 0.0673604831089 * D(EUR(-1)) - 0.0531428403778 * \\ BR2(-1) &+ 0.0338678614842 * BR1(-1) + z0.000103274192311 \end{aligned} \quad (14)$$

$$\begin{aligned} D(HOS) &= -0.0808913433303 * D(USD(-1)) + 0.117477824916 * \\ D(HOS(-1)) &+ 0.0227487183163 * GBP(-1) - 0.00574681411842 * \\ GOLD(-1) &+ 0.0748224693007 * D(EUR(-1)) + 0.0145104930665 * \\ BR2(-1) &- 0.0104834374263 * BR1(-1) - 0.000158439119473 \end{aligned} \quad (15)$$

$$\begin{aligned} \text{GBP} &= 2.91457457888 * \text{D}(\text{USD}(-1)) + 1.34297213262 * \\ &\text{D}(\text{HOS}(-1)) + 0.269532294772 * \text{GBP}(-1) + 0.389959612586 \\ &* \text{GOLD}(-1) - 1.51670883775 * \text{D}(\text{EUR}(-1)) + 0.343693439787 \end{aligned} \quad (16)$$

$$\begin{aligned} &* \text{BR2}(-1) - 0.373068778303 * \text{BR1}(-1) + 0.00428807247529 \\ \text{GOLD} &= -0.759008013788 * \text{D}(\text{USD}(-1)) + 3.59578998067 \\ &* \text{D}(\text{HOS}(-1)) - 0.00366185822931 * \text{GBP}(-1) - 0.0385987016774 \\ &* \text{GOLD}(-1) + 0.268789177917 * \text{D}(\text{EUR}(-1)) + \end{aligned} \quad (17)$$

$$\begin{aligned} &0.0962281703741 * \text{BR2}(-1) - 0.0135492419688 \\ &* \text{BR1}(-1) + 0.00319634736466 \\ \text{D}(\text{EUR}) &= 0.229634544236 * \text{D}(\text{USD}(-1)) - 0.0756726475112 \\ &* \text{D}(\text{HOS}(-1)) - 0.0176528555984 * \text{GBP}(-1) - 0.0573744919343 \\ &* \text{GOLD}(-1) - 0.0878483110903 * \text{D}(\text{EUR}(-1)) - 0.0700882483103 \end{aligned} \quad (18)$$

$$\begin{aligned} &* \text{BR2}(-1) + 0.0247709058348 * \text{BR1}(-1) + 0.000570094498376 \\ \text{BR2} &= 4.32523849455 * \text{D}(\text{USD}(-1)) + 2.69830986773 * \\ &\text{D}(\text{HOS}(-1)) - 0.0474606388638 * \text{GBP}(-1) - 0.11365371146 \\ &* \text{GOLD}(-1) - 2.34972578506 * \text{D}(\text{EUR}(-1)) - 0.269332280075 \end{aligned} \quad (19)$$

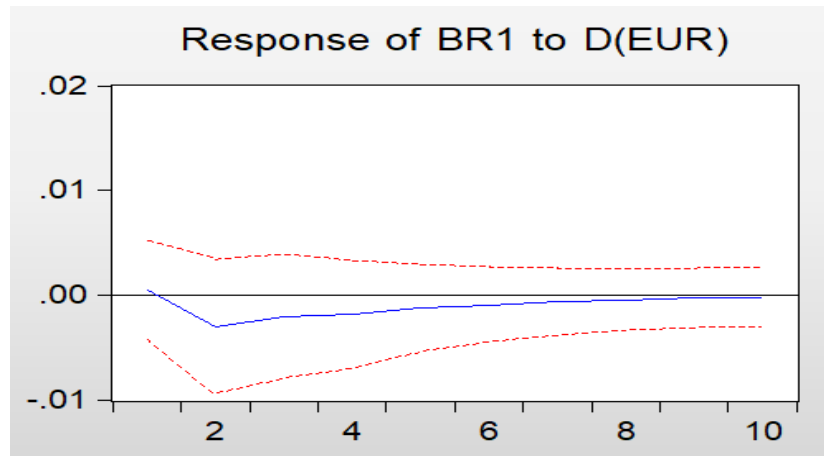
$$\begin{aligned} &* \text{BR2}(-1) + 0.850269652017 * \text{BR1}(-1) + 0.00056803155658 \\ \text{BR1} &= 2.43043273455 * \text{D}(\text{USD}(-1)) + 0.788525266412 * \\ &\text{D}(\text{HOS}(-1)) + 0.196776061841 * \text{GBP}(-1) + 0.285965364969 * \\ &\text{GOLD}(-1) - 1.58652290029 * \text{D}(\text{EUR}(-1)) - 0.164007327748 \end{aligned} \quad (20)$$

Given the literature in econometrics, estimated coefficients are neither descriptively nor politically usable for structural parameters. Thus, Action and reaction Functions are employed here. We will be looking into the results from Action and reaction Functions over the following discussion.

### Investigation and Analysis of Action and Reaction Functions

Taking into account the objective of this survey as a sample of results from Action and reaction Functions for returns on various basic economic markets, we will be discussing investigation and analysis of relationship functions between returns on different markets and those on Stock exchange. The graph below as an example indicates the impact of one unit of positive shock from Euro on Stock exchange. Evidently, one unit of positive shock from Euro leads to a decrease in the Stock exchange return. However, following several periods, this impact would converge at zero. The impact could be given an interpretation that a rise in the foreign currency market (Euro in particular) leads to a fall in the Stock exchange return. Yet, following several periods of investment in the foreign currency market, Returns on foreign currency market

sink and investors prefer not to invest there and turn to alternative markets such



as the Stock exchange.

Figure 1. RESPONSE of BR1 to D (EUR)

### Design of Scenario for Prediction Stimulation

We will be dealing with the ability of the model for out-of-sample prediction concerning Auto regression vector. To do so, the model has been estimated by 90% of the available data. This study provides the stimulation via three methods of 1) accidental 2) dynamic 3) accidental-dynamic. Accidental stimulation making the most valid prediction has been exploited in this study. Based upon this method, equations are worked out here via the accidental method. This way the formation of the model variables is accidental and is stimulated by certain probabilities. The algorithm involved in this stimulation is Broyden's Method converged optimally having 5000 times stimulation to a dot. The result from the stimulation is depicted in the graph below and out-of-sample prediction fully complies with factual observations which prove the model precision to predict the variables in the model. The second illustration shows a sample of the stimulated outcome from the model for stock exchange. As it appears the stimulation in the model has been able to explain the observations properly, it shows fewer of the dips in out-of-sample prediction and shows the point of increase in returns ahead of time which could be applied as a premature signal for return increase.

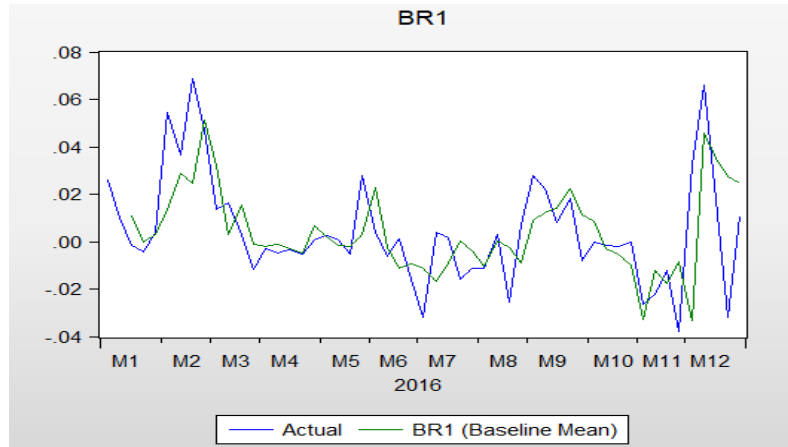


Figure 2. out-of-Sample Prediction for Stock Exchange

## Discussion and Conclusion

As discussed before there is Comonotonicity (positive or negative) with the aim of stable equilibrium between returns on basic markets of the economy. In the present study, such relationships were employed for basic markets of the economy to come up with a model and the estimation, as well as the model, were put to tests. Identification of the relationships and capability to forecast the effects of changes to returns among markets on each other do matter to policymakers in the economy as well as investors.

Having come up with optimal lag and Action/reaction Functions, we witnessed that as a consequence of one unit of fluctuation in a market along with a certain delay other markets are influenced and then the new equilibrium is established and the effect of the initial fluctuation is neutralized. The significance of the achieved results and the model mount by accepting financial-behavioral characterizations, the inefficiency of information within the framework of modern financial management and the impact by Comonotonicity. Our designed model cogently expresses the prediction of the relationships among all markets and their behavioral interaction with recognition of effectiveness lags using Comonotonic Functions under probability theory. By leading the demand of investors in various economic markets, appropriate policy-making and decision based upon maximum returns and minimum risks of investment, the outcome of the present study guides the economy to a stable equilibrium.

In the final analysis, we recommend that planners, policymakers in the



economy, managers and investors enjoy the results from the survey with emphasis on effective application of Comonotonic Functions. Given the practical abilities of the survey, we also suggest the extension of the model to other macro-markets and micro-markets controlled by various economic industries over up-to-date periods of time.

## References

- Abasinejad, Hosein, Mohamadi, Shapur (2005). Analysis of Iranian commercial cycles using Wavelet theories. *Economic Research Journal/number/75*
- Aggarwal, R., Lucey, B., O'Connor, F (2015). World metal markets. *World Scientific Handbook of Futures Markets*. Elsevier pp 385–410.
- Aggarwal, R., Soenen, L. (1988). Nature and efficiency of the gold market. *J. Portf. Manag* 4, 18–21.
- Alizadeh, Mohamad (2007). Investigation into relationships between returns on Stocks and inflation in Tehran Stock exchange over time - Various scales using Wavelet conversion. Thesis, Management faculty at Tehran University).
- Ansari, Hojatollah (2007). Investigation into effects of different time scales to calculate the value exposed to risks using Wavelet theory", Thesis, Management faculty at Tehran University)
- Batten, J.A., Ciner, C., Lucey, B.M (2014). on the economic determinants of the gold-inflation relation. *Res. Policy* 41C, 101–108.
- D.Kahneman, A.Tversky (1979). Prospect theory: an analysis of decision under risk *Econometrica* 46, 171–185.
- Fabio Bellini, Cosimo Munari, Gregor Svindland (2018). Law-invariant insurance pricing and its limitations, *Quantitative Finance*, Cornell University.
- Filis, G., Degiannakis, S., Floros, C. (2011). Dynamic correlation between the stock market and oil prices: the case of oil-importing and oil-exporting countries. *Int. Rev. Financ. Anal* (20)152–164.
- G.Carlier, R.A.Dana (2013). Pareto optima and equilibria when preferences are incompletely known, *J.Econom.Theory*148, 1606–1623.
- G.Deelstra, J.Dhaene, M.Vanmaele (2010). An overview of comonotonicity and its applications in finance and insurance, working paper.
- H.Jin, X.Y.Zhou (2008). Behavioral portfolio selection in continuous time, *Math Finance* 18) (385 426.
- H.Jin, S.Zhang, X.Y.Zhou (2011). Behavioral portfolio selection with loss control, *Acta Math.Sin*) 27 (255–274.
- J.Dhaene, M.Denuit, M.J.Goovaerts, R.Kaas, D.Vyncke (2002). The concept of comonotonicity in actuarial science and finance: theory, *Insurance Math.Econom*)31(3–33.
- J.Dhaene, M.Denuit, M.J.Goovaerts, R.Kaas, D.Vyncke (2002). The concept of comonotonicity in actuarial science and finance: applications, *InsuranceMath.Econom*) 31, 133–161.

J.DiNunno, B.Øksendal (2011). *Advanced Mathematical Methods for Finance*, Springer.

L.L.Lopes (1987). Between hope and fear: the psychology of risk, *Adv .Exp .Soc Psychol* 20 (255-295).

M.Ludkovski, L.Rüschendorf (2008). On comonotonicity of Pareto optimal risk sharing, *Statist.Probab.Lett* 78, 1181–1188

M.Kulldorff (1993), Optimal control off a vorable games with a time limit, *SIAMJ. Control Optim*31, 52–69.

Reza Tehrani and Asgar Nurbakhsh (2008). P Jones Charles, *Investment Management Translated* , 4th edition. Tehran: Negah Danesh publication.

S.Browne (1999). Reaching goals by a deadline: digital options and continuous-time active portfolio management, *Adv.in Appl.Probab*31(551–577).

S.Browne (2000). Risk constrained dynamic active portfolio management ,*Manage .Sci* 46 ,1188–1199.

Shakibaei, Alireza. Aflatuni, Abbas. Nikbakht, leili (2008). Investigation into a long-term relationship between currency exchange rate Oil prices in OPEC countries, *Science and Development Journal. Scientific Research* 15th year, no. 25, Winter 1).

Tehrani Reza. Shapur Mohamadi. Arash Mohamad Alizadeh (2011). Investigation into relationships between returns on Stocks and inflation in Tehran Stock exchange over time - Various scales using Wavelet conversion. *Summer Economic Research Journal*.

X.D.He, X.Y.Zhou (2011). Portfolio choice via quantiles, *Math.Finance*21, 203–231.

Zhang, Y., Wei, Y. (2010). The crude oil market and the gold market: evidence for co-integration, causality and price discovery. *Resour. Policy* 35, 168–177.

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