

# **A Multiscale Pricing Model with the Wavelet Analysis Approach, Fama-French Three-Factor Model, and Nonliquidity in Tehran Stock Exchange**

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## **Abstract**

*The aim of this paper is to analyze the multiscale pricing model with the wavelet analysis approach, Fama-French three-factor model, and nonliquidity in Tehran Stock Exchange. It was also desirable to figure out how stock returns, Fama-French factors, and nonliquidity were related in different intervals. According to the results, various outcomes were obtained at different intervals. Stock returns had significant relationships with  $\frac{BV}{MV}$  (the ratio of book value to market value) and nonliquidity in the long term. Stock returns had significant relationships with the beta,  $\frac{BV}{MV}$ , and company size in the midterm, too. There was also a significant relationship between stock returns and the company size in the short term. The proposed methodology suggests that investors should employ dynamic portfolio management strategy and multiscale risk-return evaluation to seize investment opportunities.*

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**JEL Classification:** G11, G19

## 1. Introduction

Investors seek to analyze the relationship between returns and risk. First, they predict the returns on every investment. Then they ask how much risk is entailed obtaining a certain level of returns. In fact, the uncertainty about future returns on stock poses risk to investment. Investment risk is the probability at which real returns occur rather than what is expected.

Classical risk analysis models such as Markowitz's model, Sharpe's single-index model, and other similar models does not help select efficient stocks and portfolios, greatly because these theories include limiting and inappreciable assumptions such as the efficiency of the market portfolio[1]. Evaluating factors affecting stock returns are a more serious problem in countries lacking an efficient stock market, because the market price of stocks is determined closely to the real value if the stock market is efficient. As a result, a multifactor model can result in the proper allocation of financial resources in the stock market by facilitating hypotheses and identifying certain factors other than the market index [2]. Such a model can finally lead to the accurate analysis of risk and stock returns at different companies, something which is the ultimate goal of forming capital markets. In 1992-1993, Fama and French indicated that other factors should also be taken into account in addition to beta (Sharpe's single-index model) in the capital asset pricing model. Fama and French studied the trend in earnings and returns at companies and analyzed the results. They concluded that there were other factors affecting the returns on stock at companies in addition to beta. According to Fama and French, either market did not act as they expected, or the capital asset pricing model was not accurate. Both cases may have also been possible. Therefore, Fama-French risk factors were used in this paper.

The liquidity of assets is another factor affecting the risk of assets. Liquidity plays a significant role in valuating assets because investors consider whether assets can turn into cash properly if they are to be sold.

According to the results of testing the above mentioned models, capital asset pricing model (CAPM) is not strong enough to determine the intervals

expected by the stock market. A CAPM defines the only factor explaining stock return difference as the systematic risk or beta coefficient. However, empirical evidence indicates that beta, regarded as the systematic risk index, is not able to explain differences in stock returns per\_se [3].

Introduced in 1980, wavelet analysis is an enhanced form of Fourier analysis. Wavelets are mathematical functions dividing data into many components (frequencies), each of which is analyzed by displaying it in a proportionate scale. An advantage of wavelets over conventional Fourier methods is their high analytical power when signals are characterized by rapid disconnections and mutations.

A wavelet filter provides a simple device to analyze the process features in a few comparisons. It is important to know that economic and financial time series do not need to follow the same relationship regarded as a function of a time horizon (scale). Thus, a wavelet divides a process into several time horizons and changes it in a way that repetitions, groups, volatile classes, fracture structures, and the general and regional characteristics of dynamism can be different in the process.

Given the fact that investors consider different investment horizons to purchase or sell securities, it is essential to analyze the relationship between returns and each of the above factors (beta,  $\frac{BV}{MV}$ , company size, and non-liquidity) entailing investment risk after all.mentioned

## **Problem Statement and Research Background**

The capital asset pricing model was created to explain how to price securities risk in the market. In fact, the CAPM is a developed version of Markowitz's modern portfolio theory. According to the CAPM, returns on every asset is the riskless rate of returns plus the net risk:

$$(R_i) = R_f + \beta_i (R_m - R_f) \quad (1)$$

Fundamental hypotheses limit the capability of the CAPM to explain and predict real returns. Fama-French three-factor model expands the capabilities of CAPM by adding two factors of special risk.

Fama and French (1992) indicated that beta could not be helpful alone, and other factors had to be taken into account. Fama and French (1993) pointed out

that the company's market and  $\frac{BV}{MV}$  had major roles in explaining differences in returns at companies [4].

Menike et al (2014), used a sample of 100 companies listed in the Colombo Stock Exchange (CSE) from 2008 to 2012 to examine the impact of dividend per share(DPS), earnings per share (EPS) and book value per share of stock price(BVPS). They used a single and multiple regression models and the results reveals that EPS, DPS, BVPS were positive and had a significant impact on the stock price in the CSE [5].

Czapkiewicz and Wojtowicz (2014), added another factor to tree-factor Fama-french model and studied the four-factor asset pricing model on the Warsaw Stock Exchange (WSE) which is one of the largest stock markets in Central and Eastern Europe. The empirical analysis is based on monthly data from the period April 2003–December 2012 which includes different stages of the business cycle. This article shows that momentum is a significant factor on the WSE and the four-factor model describes the returns variation much better than the three-factor model [6].

Duy and Phuoc pointed out that there was a negative relationship between the company size and stock returns in of Service Sector in Ho Chi Minh City Stock Exchange [7].

Sahn-WookHuh (2014), for NYSE/AMEX-listed stocks over the past 27 years estimated a set of price-impact parameters. The results show the Amihud (2002) measure is the best proxy of its kind, no low-frequency-based proxies can parallel the price-impact parameters [8]. Nonliquidity was also added to Fama-French model by the authors of this paper to expand the analysis.

It should be noted that Fama-French three-factor model was tested in several studies in Tehran Stock Exchange. For instance Eyvazlu et .al (2017) [9], Salehi et.al (2015) [10], Abbasi and Ghezljeh (2013) [11], Akbarimogaddam et.al (2009) [12], confirmed the capability of Fama-French factor model to predict stock returns in Tehran Stock Exchange.

In a paper, Vakilifard and others used two models of three-factor Fama-French and Chen model for selecting the optimum expected return. The sample was composed of 52 listed firms on the Tehran Stock Exchange for the years of 2003 to 2010 which are selected by filtering technique. The gathered data has analyzed by applying multivariate regression method. The findings reveal that the Fama-French model has higher ability in predicting the expected stock return in the capital markets [13].

Ali Norowzi has Compared Fama-French, Beta Value, and Expected Returns on Stock in Predictability and find out that there was a direct relationship between the company size and expected returns. Accordingly, there was an inverse relationship between  $\frac{BV}{MV}$  and expected returns [14].

The CAPM defines systematic risk or beta as the only factor explaining differences in returns on stock. However, empirical evidence indicated that beta, acting as the systematic risk indicator, did not have the capability to explain differences in stock returns[3]. Although most of the evidence of the relationship between returns and portfolio systematic risk confirmed CAPM, other factors such as the company size,  $\frac{BV}{MV}$ , and leverage can help describe efficiency. Many studies were conducted on the effects of these factors on returns (Bans (1981), Bandari (1988), Statman (1980), and Rosenberg Rid & Lanshtein). There is strong empirical evidence proving CAPM is wrong in real world.

Research variables and a brief account of previous studies have been presented so far. Now the wavelet analysis is described in brief. A wavelet is an assumed specific function with an average of zero. However, the fluctuations of a wavelet decrease around zero, and the function is limited in time and space. Then, an expansion is based on the traverses and dilatations of this function. The wavelet transform is a very efficient tool to deal with time series (signals) with unstable characteristics. The main idea of a wavelet transform is that it divides a time series into a series of coefficients by using a group of waveform functions. Then, these functions are created by using a basic wavelet function, also known as a mother wavelet, or analyzing wavelet. They are characterized by mathematical features such as orthogonality and unified energy.

Ramsey and Zhang (1996 & 1997) analyzed the complexity of financial data in two papers. They analyzed S&P500 data and the rates of currency exchanges to find out that financial data were not random but very complicated. Such data are characterized by great mutations acting locally. There is evidence of quasi-periodic sequences as a result of the shock imposed to the system. It appears that energy is very internal in such data [15].

Atkins and Sum (2003) used Jensen's alpha in the long-term memory and Fisher's regression in the wavelet amplitude to show that there were no

relationships between rates of interest and inflation in the short term. However, such a relationship was statistically significant in the long term [16].

Regarding Iranian studies conducted on the application of wavelet transform in financial problems, in an study, Iran's GDP was analyzed in different scales. According to the results, there were no significant differences between the wavelet transform and the Hodrick-Prescott filter in even changes of time series. However, the wavelet analysis performed better than other methods to detect cycles of time series with sudden changes [17].

Eslami employed the wavelet transform to extract time series [18] and Arash Mohammad Ali Zadeh analyzed the relationship between inflation details and the rate of monthly returns on price and cash return. Accordingly, there were only midterm relationships [19].

Hojat Ansari calculated the risk-exposed value by using the details of returns at 20 companies listed in the stock market at different levels. According to the results, the risk-exposed value was significantly smaller in longer time scales [20].

Therefore, researchers used different time scales to find relationship between the two aforesaid factors in this study.

Finally, the main research question is whether there is a relationship between Fama-French risk factors, nonliquidity and returns in different time scales.

### **Research Hypotheses**

**H<sub>1</sub>**: There is a significant relationship between beta and returns on the stock if the discrete wavelet function of maximum overlaps is used.

**H<sub>2</sub>**: There is a significant relationship between the company size and returns on the stock if the discrete wavelet function of maximum overlap is used.

**H<sub>3</sub>**: There is a significant relationship between  $\frac{BV}{MV}$  And returns on the stock if the discrete wavelet transform of maximum overlap is used.

**H<sub>4</sub>**: There is a significant relationship between returns on the stock and nonliquidity if the discrete wavelet function of maximum overlaps is used.

### **Research Methodology**

Monthly returns and book values of companies were extracted from Rahavard Novin. All of the data such as final price, the number of listed stocks, market value, Rial value of transactions, and the number of transactional days were extracted from [www.TSETMC.com](http://www.TSETMC.com).

#### Statistical Population and Sample

The statistical population included the 40 companies listed in Tehran Stock Exchange. Then the following constraints were placed on the sample:

1. The fiscal years of companies should end on March 20.
2. The stocks of companies should exceed 100 days of transaction every year.
3. Companies should not act as financial intermediaries.

#### Research Domain

This study was conducted on 40 companies that listed in Tehran Stock Exchange from 2012 to 2017.

#### Data Analysis

The company size,  $\frac{BV}{MV}$ , beta, and monthly liquidity were obtained from the statistical sample. Then, monthly portfolios were created.

Riskless return ( $R_f$ ): the rate of returns on Islamic Treasury bills issued by Iran's Government. We used average rate of this bills equal with 20.30%.

Return: the final price of stocks is based on the increased capital (obtained from demands and cash brought by the accumulated earnings), cash earnings, and awarded stocks. Then, stock returns are calculated.

The balanced prices are used to calculate returns:

$$r_{i,t+1} = \frac{(1+x+y)P_{i,t+1} - P_{i,t} - y \times 1000 + D}{P_{i,t} + 1000xy} \quad (2)$$

$r_{i,t+1}$ : the return rate of stock  $i$  in  $t+1$

$x$ : the percentage of increased capital in savings

$y$ : the percentage of increased capital in cash and demands

$P_{i,t+1}$ : the price of every stock  $i$  at the end of  $t$

$P_{i,t}$ : the price of every stock  $i$  at the beginning of  $t$

D: dividend earnings of stock  $i$

1000: the nominal value of every stock or the registration price of every stock

Riskless return is subtracted from the resultant returns to obtain the net return<sup>1</sup>. The final balanced daily price and the total index of Tehran Stock Exchange (2012-2017) were used to estimate the monthly beta of every stock. The monthly return of every stock is based on the final transactional prices of every stock in every month. If no transactions are conducted on the stock, the return of that month will be considered zero.

First, the index return and daily return of stocks are calculated via  $(\frac{P_1}{P_0}) - 1$ , then, the covariance of the total index returns and the stock return is divided by the variance of the total stock return. The resultant number indicates the stock beta in a month.

Liquidity refers to the ability to purchase and sell stocks at the lowest cost in the shortest possible time [21].

In this study, Amihud's non-liquidity index was used. According to this index, the absolute value of monthly returns was divided by the monthly real value of transactions. The resultant number was then divided by the number of transaction days in a month.

Portfolios were created by using the company size and  $\frac{BV}{MV}$ . In other words, companies were classified as large and small (these two groups can be separated by a median). The companies were also classified as small, medium, and large by  $\frac{BV}{MV}$  (The separation boundary was considered the 30<sup>th</sup> and 70<sup>th</sup> percentiles). Therefore, SMB and HML were obtained.

The monthly beta was obtained by assuming equal weights of the sample companies. Their rates of return should be calculated to obtain SMB, HML, and ILM. Therefore, ILM return is obtained from the difference between companies having 30% of higher liquidity and 30% of lower liquidity.

SMB returns are calculated by determining the difference in the average return between large and small companies.

HML returns are obtained by determining the difference in the average return between companies with higher  $\frac{BV}{MV}$  and companies with lower  $\frac{BV}{MV}$ .

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<sup>1</sup> Monthly returns were extracted from Rahavard Novin.



According to the results of testing hypotheses and the null hypothesis ( $H_0: \rho=0$ ), the correlation coefficient of different factors is compared with returns in different scales.

After forming SMB and HML, there are six portfolios with weights based on value, size intersection, and  $\frac{BV}{MV}$  at the end of each month. They are referred to as SL, SM, SH, BL, BM, and BH. The returns of relevant companies are referred to as  $R_{SL}$ ,  $R_{SM}$ ,  $R_{SH}$ ,  $R_{BL}$ ,  $R_{BM}$ , and  $R_{BH}$ .

SMB is obtained from the difference in the average monthly mean of three small portfolios and the average monthly mean of three large portfolios:

$$SMB = \bar{R}_S - \bar{R}_B = \frac{1}{3}[R_{SL} + R_{SM} + R_{SH}] - \frac{1}{3}[R_{BL} + R_{BM} + R_{BH}] \quad (3)$$

Thus, there will be one SMB per month.

HML is obtained from the difference in the average monthly mean of two portfolios of higher  $\frac{BV}{MV}$  and the average monthly mean of two portfolios of

lower  $\frac{BV}{MV}$ :

$$HML = \bar{R}_H - \bar{R}_L = \frac{1}{2}[R_{SH} + R_{BH}] - \frac{1}{2}[R_{SL} + R_{BL}] \quad (4)$$

After obtaining beta, the wavelet transform was applied to SMB, HML, and ILM in MATLAB.

### Research Findings

Considering the number of data (72), analysis of information was done in six levels:

$$2^6 = 64 \qquad 64 < 72 < 128 \qquad 2^7 = 128$$

$$s = a_6 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1 \quad (5)$$

S: the main time series

$a_6$ : level estimation component

$d_6, d_5, d_4, d_3, d_2, d_1$ : components of details at levels 1-6

At level  $j$ , the scale is  $2^j$ , and the resolution is obtained from the following formula:

$$a = 2^j, \text{ resolution} = \left(\frac{1}{a}\right) \times N \quad (6)$$

Accordingly, in our wavelet analysis we use six time intervals with different levels are as follows:

The first level: 36 months (1080 days)

The second level: 18 months (540 days)

The third level: 9 months (270 days)

The fourth level: 4 and a half months (135 days)

The fifth level: 2 months and 8 days (nearly 68 days)

The sixth level: nearly one month (34 days)

The first level of analysis (1080 days), the third level of analysis (270 days), and the sixth level of analysis (34 days) were regarded as the long term, the midterm, and the short term in this study.

The following table shows the results of the first level ( $d_1$ ):

**Table 1. Correlation Coefficient of Variables at the First Level**

|        | Return          | Beta    | SMB             | HML     | ILM     |
|--------|-----------------|---------|-----------------|---------|---------|
| Return | 1               | 0.0483  | $-6.8211e^{-4}$ | -0.3278 | -0.3595 |
| Beta   | 0.0483          | 1       | 0.0070          | 0.0852  | -0.0906 |
| SMB    | $-6.8211e^{-4}$ | 0.0070  | 1               | 0.4137  | -0.7138 |
| HML    | -0.3278         | 0.0852  | 0.4137          | 1       | -0.2590 |
| ILM    | -0.3595         | -0.0906 | -0.7138         | -0.2590 | 1       |

**Table 2. The Probability of Significant Variables at the First Level**

|        | Return | Beta   | SMB            | HML    | ILM            |
|--------|--------|--------|----------------|--------|----------------|
| Return | 1      | 0.7583 | 0.9965         | 0.0319 | 0.0179         |
| Beta   | 0.7583 | 1      | 0.9644         | 0.5870 | 0.5635         |
| SMB    | 0.9965 | 0.9644 | 1              | 0.0058 | $7.7333e^{-8}$ |
| HML    | 0.0319 | 0.5870 | 0.0058         | 1      | 0.0935         |
| ILM    | 0.0179 | 0.5635 | $7.7333e^{-8}$ | 0.0965 | 1              |

Accordingly, there is a significant relationship between HML and ILM at the long term analysis because  $0.05 > 0.0319$  and  $0.05 > 0.01790$ .

At the third level ( $d_3$ ):

**Table3. Correlation Coefficient of Variables at the Third Level**

|        | Return  | Beta    | SMB     | HML     | ILM     |
|--------|---------|---------|---------|---------|---------|
| Return | 1       | -0.4884 | -0.5999 | -0.7117 | 0.2260  |
| Beta   | -0.4884 | 1       | 0.8944  | 0.7257  | 0.0024  |
| SMB    | -0.5999 | 0.8944  | 1       | 0.8765  | -0.2742 |
| HML    | -0.7117 | 0.7257  | 0.8765  | 1       | -0.4259 |
| ILM    | 0.2260  | 0.0024  | -0.2742 | -0.4259 | 1       |

**Table4. The Probability of Significant Variables at the Third Level**

|        | Return         | Beta           | SMB            | HML            | ILM    |
|--------|----------------|----------------|----------------|----------------|--------|
| Return | 1              | 0.0211         | 0.0032         | $2.0351e^{-4}$ | 0.3118 |
| Beta   | 0.0211         | 1              | $2.0052e^{-8}$ | $1.3207e^{-4}$ | 0.9915 |
| SMB    | 0.0032         | $2.0052e^{-8}$ | 1              | $8.8424e^{-8}$ | 0.2169 |
| HML    | $2.0351e^{-4}$ | $1.3207e^{-4}$ | $8/8424e^{-8}$ | 1              | 0.0481 |
| ILM    | 0.3118         | 0.9915         | 0.2169         | 0.0481         | 1      |

Accordingly, the return had significant relationships with beta, SMB, and HML at midterm structure:  $0.05 > 0.0211$ ;  $0.05 > 0.0032$ ;  $0.05 > 0.00020$

At the sixth level ( $d_6$ ):

**Table3. Correlation Coefficients of Variables at the Sixth Level**

|        | Return         | Beta    | SMB     | HML            | ILM     |
|--------|----------------|---------|---------|----------------|---------|
| Return | 1              | 0.2000  | -0.7923 | $6.6304e^{-4}$ | -0.4094 |
| Beta   | 0.2000         | 1       | -0.7247 | 0.6466         | -0.4684 |
| SMB    | -0.7923        | -0.7247 | 1       | -0.3731        | 0.6252  |
| HML    | $6.6304e^{-4}$ | 0.6466  | -0.3731 | 1              | -0.5571 |
| ILM    | -0.4094        | -0.4684 | 0.6252  | -0.5571        | 1       |

**Table 6. The Probability of Significant Variables at the Sixth Level**

|        | Return         | Beta   | SMB            | HML    | ILM    |
|--------|----------------|--------|----------------|--------|--------|
| Return | 1              | 0.4747 | $4.2905e^{-4}$ | 0.9981 | 0.1296 |
| Beta   | 0.4747         | 1      | 0.0022         | 0.0092 | 0.0782 |
| SMB    | $4.2905e^{-4}$ | 0.022  | 1              | 0.1707 | 0.0127 |
| HML    | 0.9981         | 0.0092 | 0.1707         | 1      | 0.0310 |
| ILM    | 0.1296         | 0.0782 | 0.0127         | 0.0310 | 1      |

Accordingly, there was only one significant relationship between return and SMB ( $0.05 > 0.000420$ ) at short term.

### Conclusion and Suggestions

**The First Result:** There was a significant and an inverse relationship between returns and  $\frac{BV}{MV}$  in the long term. At the same time, there was also an inverse and significant relationship between returns and non-liquidity. However, there were no significant relationship between returns on bet and size. In comparison Duy and Phuoc pointed out that there was a negative relationship between the company size and stock returns in of Service Sector in Ho Chi Minh City Stock Exchange in long term.

**The Second Result:** There was an inverse and significant relationship between return and beta in the midterm. Likewise, there was an inverse and significant relationship between return and  $\frac{BV}{MV}$ . However, there were no significant relationship between the return and nonliquidity in the midterm.

**The Third Result:** There was an inverse and significant relationship between return and company size in the short term. There was also a significant relationship between return and three other factors (beta,  $\frac{BV}{MV}$ , and non-liquidity).

In comparison Ali Norowzi point out there was a direct relationship between the company size and expected returns. Accordingly, there was an inverse relationship between  $BV/MV$  and expected returns in Tehran stock but He didn't compare the relationships in different time intervals.

Overall, the results of our research shows:

1. There was a significant relationship between stock returns and beta in the midterm by using the discrete function of maximum overlap.

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2. There was an inverse relationship between stock returns and company size in the short term by using the discrete function of maximum strap. The relationship was inverse in the midterm, too. There was no relationship in the long term.
  3. The relationship between stock returns and  $\frac{BV}{MV}$  was not significant in the short term by using the discrete function of maximum overlap. This relationship was inverse in both midterm and long term.
  4. There was not a significant relationship between stock returns and non-liquidity in the short term and midterm by using the discrete function of maximum overlap. In the long term, this relationship was significant and inverse.

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