

Comparing Different Models of Evolutionary Three-Objective Optimization Using Fuzzy Logic in Tehran Stock Exchange

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Abstract

Optimal Portfolio Selection is one of the most important issues in the field of financial research. In the present study, we try to compare four various different models, which optimize three-objective portfolios using “Postmodern Portfolio Optimization Methods”, and then to solve them. These modeling approaches take into account both multidimensional nature of the portfolio selection problem and requirements imposed by investors. Concretely different models optimize the expected return, the down side risk, skewness and kurtosis given portfolio, taking into account budget, bounds and cardinality constrains. The quantification of uncertainty of the future returns on a given portfolio is approximated by means of LR-fuzzy numbers, while the moments of its returns are evaluated using possibility theory. In order to analyze the efficient portfolio, which optimize three criteria simultaneously, we build a new NSGAI algorithm, and then find the best portfolio with most Sortio ratio from the gained Pareto frontier. Thus, in this paper we choose 153 different shares from different industries and find their daily return for ten years from April of 2006 till March of 2017 and then we calculate their monthly return, downside risk, skewness, kurtosis and all of their fuzzy moments. After designing the four models and specific algorithm, we solve all of the four

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models for ten times and after collection of a table of the answers, compare all of them with Treynor ratio. At last, we find that using fuzzy and possibilistic theory make higher return and better utilized portfolios.

Key words: Modern Portfolio Theory, Post Modern Portfolio Theory, Financial Modeling, Optimize Portfolio Selection, Evolutionary Multi-Objective Algorithm, Fuzzy Logic

1. Introduction

Optimal Portfolio Selection is one of the most important issues in the field of financial research. This means what combination of assets the investor should choose to maximize the utilization with relevant limitation. Markowitz considered returns as a random variable and solved this problem with maximization of return and minimization of variance as risk criteria. However, return on securities in the real world is usually vague and inaccurate. In today's world of investment, one of the challenges of investment in return on assets is uncertainty of future events and their consequences.

Solving the problem of selecting portfolios requires two key components: a) A suitable method for quantitatively measuring the uncertainty of future returns of the desired portfolio; and b) The optimization procedure and method that can provide Pareto optimal portfolios that meet the investors' requirements (D.Dubois, Prade, 1980)

In the recent years, there has been a growing interest of including information about trading and investors' requirements into the portfolio selection problem, since not all the relevant information for portfolio selection can be obtained just by optimizing returns and risk simultaneously. Thus, some practical constraints have been added to portfolio selection problem in order to make it more realistic, such as upper and lower bound constrains, allowing asset combinations which respect the investors' wishes, or cardinality constrains limiting the number of assets participating in the portfolios. With the introduction of such constraints, the portfolio optimization problem becomes a constrained multi-objective problem that is NP-hard, and traditional optimization methods cannot be used to find efficient portfolios (Reben Saborido, Ruiz, Bermudez, Vercher, 2016).

The main purpose of this paper is to solve the problem of portfolio selection model by three goals optimizing simultaneously. Therefore, we design two different models in two different ways of fuzzy and simple (non-fuzzy) for

simultaneously optimization of three-objective modelling, by using mean, downside risk, skewness and kurtosis. After that, we design a new method of NSGAI algorithm to solve models and finally, we conclude that using fuzzy logic has a positive effect on portfolio optimization process and using skewness in three-objective modeling can make much better portfolios than using kurtosis.

1. Formulation and Background Concepts

Multi-objective optimization problems are mathematical programming problems with a vector-valued objective function, which is usually denoted by $f(x) = (f_1(x), \dots, f_n(x))$ for a decision vector $x=(x_1, \dots, x_N)$, where $f_j(x)$ is a real-valued function defined over the feasible set $S \subseteq \mathbb{R}^N$, for every $j=1, \dots, n$. Consequently, the decision space belongs to \mathbb{R}^N , while the criterion space belongs to \mathbb{R}^n , and the multi-objective optimization problem can be stated as follows:

$$\begin{aligned} \text{Optimize: } & \{f_1(x), \dots, f_n(x)\} & (1) \\ \text{s.t. } & x \in S. \end{aligned}$$

In the criterion space, some objective functions must be maximized ($j \in J_1$) while others must be minimized ($j \in J_2$), where J_1 and J_2 verify that $J_1 \cup J_2 = \{1, \dots, n\}$ and $J_1 \cap J_2 = \emptyset$.

We say that $x \in S$ is Pareto optimal or efficient solution if there does not exist another $x' \in S$, such that $f_j(x') \geq f_j(x)$ for every $j \in J_1$ and $f_j(x') \leq f_j(x)$ for every $j \in J_2$. with at least one strict inequality. The set of all Pareto optimal solutions $x \in S$ (in the decision space) is called the Pareto optimal set, denoted by E , and the set of all of their corresponding objective vectors $f(x)$ (in the criterion space) is called the Pareto optimal front, denoted by $f(E)$. Additionally, given two objective vectors $z, z' \in \mathbb{R}^n$, we say that z' dominated z if and only if $z'_j \leq z_j$ for every $j \in J_1$ and $z_j \leq z'_j$ for every $j \in J_2$ with at least one strict inequality. If z and z' do not dominate each other, they are said to be non-dominated (Reben Saborido, Ruiz, Bermudez, Vercher, 2016).

In the multi-objective optimization, the ideal and the nadir points are very useful since they define lower and upper bounds for the objective functions,

respectively. The ideal point $z^* = (z_1^*, \dots, z_n^*)^T \in \mathbb{R}^n$ is obtained by optimizing each objective function individually over the feasible set, that is $z_j^* = \max_{x \in S} f_j(x) = \min_{x \in S} f_j(x)$ for every $j \in J_1$ and $z_j^* = \max_{x \in S} f_j(x) = \min_{x \in S} f_j(x)$ for all $j \in J_2$. The nadir point $z^{nad} = (z_1^{nad}, \dots, z_n^{nad})^T \in \mathbb{R}^n$ is defined as $z_j^{nad} = \min_{x \in S} f_j(x)$ for all $j \in J_1$ and $z_j^{nad} = \max_{x \in S} f_j(x)$ for all $j \in J_2$. In practice, the nadir point is difficult to calculate and the general practice is to estimate it with different approaches [38, 39]. Sometimes, a vector that dominates every Pareto optimal solution is required, which means that it is strictly better than the ideal point. This vector is denoted by $z^{**} = (z_1^{**}, \dots, z_n^{**})^T$ and it is called a utopian point. In practice z^{**} can be defined by $z_j^{**} = z_j^* + \varepsilon_j$ for all $j \in J_1$ and $z_j^{**} = z_j^* - \varepsilon_j$ $j \in J_2$, where $\varepsilon_j > 0$ is a small real number for every $j = 1, \dots, n$ (Reben Saborido, Ruiz, Bermudez, Vercher, 2016).

The use of evolutionary multi-objective optimization (EMO) algorithm for solving multi-objective optimization problems has become very popular in the last two decades and, currently it is one of the most active research fields [3.4]. In the EMO field, solving a multi-objective optimization problem is understood as finding a set of non-dominated solution as close as possible to Pareto optimal front (diversity). EMO algorithms are population-based approaches which start with randomly created population of individuals. Afterwards, the algorithm enters in an iterative process that creates a new population at each generation, by the use of operators which simulates the process of natural evolution: selection, crossover, mutation and/or elitism preservation. One of the main advantages of EMO algorithms is that they are very versatile, as they can deal with multi-objective optimization problems having variables and objective functions of different nature (X. Zou, Chen, Liu, Kang, 2008).

One of the most used EMO algorithms is the Non-dominated Sorting Genetic Algorithm (NSGAII) (K. Deb, Pratap, Agarwal, 2002). This algorithm is based on the Pareto dominance and has stood out by its fast non-dominated sorting procedure for ranking solutions and its elite-preserving mechanism for the selection of best individuals. Starting from a randomly generated population, at each generation of NSGAII, and offspring population is created through selection, cross over and mutation operators. Then both parents and offspring are joined and classified into non-dominated fronts as follow. From the resulting population, the individuals who are not dominated by any other

solution constitute the so-called first non-dominated front. These individuals are temporarily removed and, subsequently, the second non-dominated front is formed by next individuals, who are not dominated by any other solution in the population. This process continues until every individual has been included into a single front. Afterwards, the population to be passed to the next generation is formed by the solutions in the lower level of non-dominated fronts.

2. Previous related works

While classical portfolio optimization problems can be efficiently solved by applying classical optimization techniques, this is not the case if additional conditions, such as diversification and cardinality constraints, are introduced. Mainly, the most significant difficulty is the generation of feasibility portfolios satisfying the requirements imposed by investors. Additionally, the solution process required for finding the set of efficient portfolios is not trivial. Note that the management of non-dominated solution is computationally simple in the bi-objective case, but it is much complicated in presence of multi-objective (three or more). In this regard, the usefulness of evolutionary multi-objective optimization for solving the constrained multi-objective portfolio selection problem is doubtless because of its ability to handle multiple criteria and constraints at the same time (K. Liagkouras, Metaxiotis, 2015). The reason for such a success is that they work with the problem as a black box, only considering inputs and outputs, without requiring additional information like derivation or continuity of properties of objective functions (Khajezadeh Dezfuli, Mehdi, 2016).

For example, Chang et al. (T.J.Chang, Meade, Beasley, Sharaiha, 2000) applied these types of algorithms to the mean-variance (MV) model. They showed that limiting the number of assets in portfolio (i.e including cardinality constraints) and considering lower and upper bounds for budget invested in such assets modify the shape of Pareto optimal front. They demonstrated that in the presence of constraints, the Pareto optimal front of MV model is more difficult to approximate, because it may become discontinuous.

To handle cardinality constraints in the MV bi-objective optimization model, a few hybrid strategies have been applied quite often. In (D.Maringer, Kellerer, 2003), a combined simulated annealing and evolutionary optimization

procedure was suggested, while Moral-Escudero et al. (R.Moral-Escudero, Ruiz-Torrobiano, Suarez, 2006) used a hybrid strategy that made use of genetic algorithms and quadratic programming for selecting the optimal subset of assets. In (R.Moral-Escudero, R.Ruiz-Torrobiano, A.Suarez, 2007), an EMO algorithm was combined with fuzzy logic in order to facilitate the trade-off between the two objectives. They also analyzed the performance of several EMO algorithms for solving the constrained MV portfolio optimization problem. For solving the cardinality constrained MV model, Chiam et al. (S.C.Chiam, Tan, Mamum, 2008) proposed an order-based representation with suitable variation operator and some techniques for handling constraints. More recently, Anagnostopoulos and Mamanis (K.P. Anagnostopoulos, G.Mamanis, 2008), applied three EMO algorithms in order to explore the Pareto optimal Front if cardinality was included into the MV model as an additional objective to be minimized. Also, Anagnostopoulos and Mamais (S.C.Chiam, Tan, Mamum, 2008) presented a study in which five EMO approaches were applied to the cardinality constrained MV model for solving instances of data sets with large number of assets, showing a clear superiority of the evolutionary algorithms SPEA2(Zitzler, M.Laumanns, L.Thiele, 2001) and NSGAI (K.Deb, Pratap, Agarwal, Meyarivan, 2002). Lately, Liagkouras and Metaxiotis (K.Liagkouras, Metaxiotis, 2014) have proposed a mutation operation for solving the cardinality constrained MV model, which was compared with the classical polynomial mutation operator also in NSGAI and SPEA2.

Other authors solve portfolio selection models with the alternative measure of risk and/or additional constraints by means of EMO algorithms. For example, Chang et al. (T.J.Chang, Yang, Chang, 2009) applied a genetic algorithm for solving bi-objective optimization portfolio selection problem with different risk measures (variance, semi-variance, absolute deviation), including skewness as a constraint. These authors reported the fact that investors should not consider portfolio sizes above one third of the total number of assets, because they are dominated by other portfolios with less positive components.

Note that the majority of the aforementioned evolutionary procedures are applied for solving extension of MV model, in which the expected return and risk of assets are considered as known parameters. Instead, fuzzy portfolio selection approaches allow the use of fuzzy numbers either for representing the investor's aspiration level for expecting return and risk, or for quantifying the

uncertainty of future returns on assets, using credibility or possibility distribution. For example Bhattacharyya et al. (R.Bhattacharyya, Kar, Majumder, 2011) considered a fuzzy mean-variance-skewness model with cardinality and trading constraints, which is solved by applying both fuzzy simulation and an elitist optimization genetic algorithm. Bernidez et al. (D.Bermudez, Segura, Vercher, 2012) implemented a bi-objective optimization genetic algorithm for solving a fuzzy mean-downside risk portfolio selection problem with cardinality constraints and diversification conditions, in which the approximation of the uncertain returns was done through trapezoidal fuzzy numbers. In (P.Gupta, Inuiguchi, Mehlawat, Mittal, 2013), a multi-criteria credibilistic portfolio selection model was proposed, which maximized (short and long-term) return and liquidity and considered the portfolio risk as a credibility-based fuzzy chance constraint. The fuzzy estimates were obtained assuming both trapezoidal possibility distributions and general functional forms. This model, which also included budget, bound and cardinality constraints was solved by applying a hybrid algorithm that integrated fuzzy simulation with a real-coded genetic algorithm.

2.1.Fuzzybackground

In the portfolio optimization problem, the modeling of uncertain returns on assets is made using different approaches. Some authors assume that these returns are random variables, while others consider fuzzy logic to integrate the uncertainty of the datum, imperfect knowledge of market behavior, imprecise investors' aspirations levels and experts opinion and so on. However, expected return on asset modeling either as random variable or fuzzy quantities are considered as known parameters of optimization problem, which have been usually estimated throughout historical data set (Reben Saborido, Ruiz, Bermudez, Vercher, 2016).

In the MDRS model (Enriqueta.Vercher, Bermudez, 2013), fuzzy numbers are considered to present the uncertainty of the investment return on a given portfolio and historical data information is used for building suitable membership functions for returns of the investment. For a capital market with N financial assets, a portfolio of $x=(x_1, \dots, x_N)^T$ determines how the total wealth of investors is allocated, where x_i is the function of the total investment devoted to asset i for every $i=1, \dots, N$. In this approach, its return over T period,

denoted by $\{r(x)\}_{t=1}^T$, are used as the historical data set, and the possibility of distribution of return on x allows us to evaluate their corresponding interval-mean valued expectations. Let us briefly review some definitions and results required about fuzzy set theory (D.Dubois, Prade, 1978, 1980, M.Inuiguchi, T.Tanino,2002).

A fuzzy number Q is said to be an LR-type fuzzy number if its membership function has the following form:

$$\mu_Q(y) = \begin{cases} L\left(\frac{A-y}{s_A}\right) & \text{if } -\infty < y \leq A, \\ 1 & \text{if } A \leq y \leq B, \\ R\left(\frac{y-B}{s_B}\right) & \text{if } B \leq y < +\infty, \end{cases}$$

Where A and B satisfy $A \leq B$, and represent the lower and upper bounds of the core of Q representatively, i.e., $\{y | \mu_Q(y) = 1\} = [A, B]$ that s_A and s_B are the left and the right spread of Q and $L, R: [0, +\infty) \rightarrow [0, 1]$ are reference functions which are non-increasing and upper semi-continuous with $\lim_{t \rightarrow \infty} L(t) = \lim_{t \rightarrow \infty} R(t) = 0$. A fuzzy number Q is said to be bounded LR-type fuzzy number if the reference functions are such that the support of Q is bounded, i.e., if there exist two real numbers a and b , with $a < b$, such that $\{y: \mu_Q(y) > 0\} \subset [a, b]$.

For a portfolio x , let us consider the reference function $L_\pi(t) = 1 - t^\pi$ and $R_\rho(t) = 1 - t^\rho$, where π and ρ are their positive shape parameters, respectively, for every $t = 1, \dots, T$. Thus, we have a power LR-fuzzy number Q induced a possibility distribution that matches with its membership function $\mu_Q(y)$ (L.A.Zadeh, 1978), we consider power LR-fuzzy number to approximate the uncertain return on the portfolio x , and we directly approximate the possibility distribution of its return instead of aggregating the possibility distribution of the individual assets that compose x . Then, to approximate the expected rerun on a given portfolio, the concept of interval-valued expectation (L.A.Zadeh, 1978) is applied, with the usual defuzzification approach for a crisp represent of their possibility moments. It must be

mentioned that other weighted mean-interval definitions could be used analogously (R.Fullér, Majlender, 2003).

Let us introduce the explicit formulation of possibilistic moment considered in (Enriqueta Vercher, Bermudez, 2013) as objective functions of the portfolio selection MDRS model:

$$\bar{E}(Q) = \frac{p_u + p_l}{2} + \frac{d}{2} \frac{\rho}{\rho + 1} - \frac{c}{2} \frac{\pi}{\pi + 1}$$

First possibilistic
moment: Possibilistic
mean value

$$w(Q) = p_u + p_l + d \frac{\rho}{\rho + 1} + c \frac{\pi}{\pi + 1}$$

Second possibilistic
moment:
Possibilistic
downside risk

$$\begin{aligned} \mu_3(Q) = & \frac{1}{4} \left(d \frac{\rho}{\rho + 1} - c \frac{\pi}{\pi + 1} \right)^3 + \frac{1}{2} \left(d^3 \frac{\rho}{\rho + 3} - c^3 \frac{\pi}{\pi + 3} \right) + \frac{3(p_u - p_l)}{4} \\ & \times \left[d^2 \left(\frac{\rho}{\rho + 2} - \frac{\rho^2}{(\rho + 1)^2} \right) - c^2 \left(\frac{\pi}{\pi + 2} - \frac{\pi^2}{\pi + 1^2} \right) \right] \\ & - \frac{3}{4} \left(d^2 \frac{\rho}{\rho + 2} + c^2 \frac{\pi}{\pi + 2} \right) \left(d \frac{\rho}{\rho + 1} - c \frac{\pi}{\pi + 1} \right) \end{aligned}$$

Third possibilistic
moment:
Possibilistic
Skewness

$$\begin{aligned} K(\tilde{x}) = & - \left[\frac{1}{4} (l_s(\tilde{x}) + l_c(\tilde{x})) \right]^5 \left(\frac{|\alpha - \beta|}{5 \alpha \beta} \right) + \max \left(\frac{\left(\frac{|\alpha - \beta|}{4} - \frac{1}{2} l_c(\tilde{x}) \right)^5}{10 \alpha \cup \beta}, 0 \right) \\ & + \frac{\left(\frac{|\alpha - \beta|}{4} + \frac{1}{2} l_c(\tilde{x}) \right)^5 |\alpha - \beta|}{10 \alpha \cup \beta} \frac{1}{2 \alpha \beta} \left[\frac{1}{2} (l_s(\tilde{x})) \right. \\ & \left. - \frac{(\alpha + \beta)}{4} \right] \left[\frac{1}{4} (l_s(\tilde{x}) + l_c(\tilde{x})) \right]^4 - \frac{\left(\frac{|\alpha - \beta|}{4} + \frac{1}{2} l_c(\tilde{x}) \right)^5}{10 \alpha \cap \beta} \end{aligned}$$

Forth possibilistic
moment:
Possibilistic
Kurtosis

3. Modeling

In this section, we describe several models that we have used to determine the best portfolio selection, and after that, we will solve the models, compare the results, and rank them.

Let us consider a capital market with N financial assets offering uncertain rates of returns. As the investor desires to know which optimal allocation of their wealth exists among the N assets, the maximization of investment return at the end of the period is looked for. Let us consider a portfolio $x=(x_1, x_2, \dots, x_N)^T$ in which the total wealth is allocated, where x_i is a fraction of the total investment devoted to the asset i , for every $i=1, \dots, N$. The portfolio must verify that

$$\sum_{i=1}^N x_i = 1$$

And the non-negative condition of every proportion, $x_i \geq 0$ for every $i=1, \dots, N$ when short selling is excluded. In the models, the expected return on asset i and its possibilistic moments are not considered as known parameters. The possibility distribution of a given portfolio is directly approximated instead of using aggregation of the possibility distribution of the individual assets, as mentioned before. According to this, the return of portfolio x is modeled by the power of LR-fuzzy number, denoted by $P_x = (p_l, p_u, c, d)_{L, \pi, R, \rho}$ whose cord and spreads are directly defined as functions of suitable percentile q_j 's, while the shape parameters are evaluated using the preserve rating procedure, using those percentiles with a 50% possibility of being realistic. The modeling approach has been previously used for fuzzy ranking risky portfolios and for obtaining efficient portfolios using bi-objective optimization procedure (Vercher, Enriqueta Bermudez, 2015). For building efficient portfolios, in different models, we take into account the simple return, possibilistic expected return, semi-deviation, possibilistic semi-deviation below mean, skewness, possibilistic skewness, kurtosis and possibilistic possibilistic in four different models (Reben Saborido, Ruiz, Bermudez, Vercher, 2016).

Additionally, we imposed limits on the budget to be invested in each asset i by using lower and upper bounds, denoted by l_i and u_i , respectively, for every $i=1, \dots, N$. Besides, in order to control the number of assets in the portfolio, an additional constraint is incorporated to assure that the number of non-negative components in each portfolio is within an interval $[k_l, k_u]$, for given values k_l and k_u .

Based on aforementioned assumptions, the cardinality constrained, we design four different models as follow:

(1) fuzzy mean, fuzzy downside risk, fuzzy skewness

$$\max E(\tilde{P}_X) \quad (1)$$

$$\min W(\tilde{P}_X) \quad (2)$$

$$\max S(\tilde{P}_X) \quad (3)$$

St

$$\sum_{i=1}^N x_i = 1 \quad (4)$$

$$k_l \leq c(x) \leq k_u \quad (5)$$

$$0 \leq l_i \leq x_i \leq u_i \quad (6)$$

$$\text{for } i = 1, 2, \dots, n$$

That $\tilde{\mu}_x$ is possibilistic expected return, $\mu_x = E\{\tilde{P}_X\}$ possibilistic semi-deviation below mean is $W(\tilde{P}_X)$, $S(\tilde{P}_X)$ is the possibilistic skewness.

(2) fuzzy mean, fuzzy downside risk, fuzzy kurtosis

$$\max E(\tilde{P}_X) \quad (7)$$

$$\min W(\tilde{P}_X) \quad (8)$$

$$\max K(\tilde{P}_X) \quad (9)$$

St

$$\sum_{i=1}^N x_i = 1 \quad (10)$$

$$k_l \leq c(x) \leq k_u \quad (11)$$

$$0 \leq l_i \leq x_i \leq u_i \quad (12)$$

for $i = 1, 2, \dots, n$

That \tilde{P}_x is possibilistic expected return, $W(\tilde{P}_x)$ is the possibilistic semi-deviation below mean, and $K(\tilde{P}_x)$ is the possibilistic kurtosis.

(3) mean, downside risk, skewness

$$\max E(P_x) \quad (13)$$

$$\min W(P_x) \quad (14)$$

$$\max S(P_x) \quad (15)$$

St

$$\sum_{i=1}^N x_i = 1 \quad (16)$$

$$k_l \leq c(x) \leq k_u \quad (17)$$

$$0 \leq l_i \leq x_i \leq u_i \quad (18)$$

for $i = 1, 2, \dots, n$

That P_x is the expected return, $W(P_x)$ is the semi-deviation below mean, and $S(P_x)$ is the possibilistic skewness.

(4) mean, downside risk, kurtosis

$$\max E(P_x) \quad (19)$$

$$\min W(P_x) \quad (20)$$

$$\max K(P_x) \quad (21)$$

St

$$\sum_{i=1}^N x_i = 1 \quad (22)$$

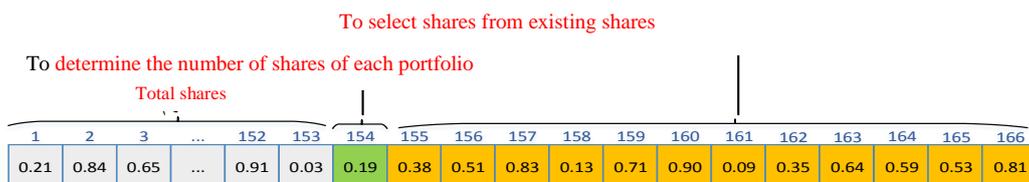
$$k_l \leq c(x) \leq k_u \quad (23)$$

$$0 \leq l_i \leq x_i \leq u_i \quad (24)$$

for $i = 1, 2, \dots, n$

4. Portfolio selection problem

Clearly, the creation of a suitable structure for the chromosome in a multi-objective genetic algorithm can have a significant effect on the quality and efficiency of this algorithm. In fact, a suitable structure for the chromosome can lead to a full space search, and the result of the algorithm will be a better answer. What makes it difficult to use this algorithm in this problem is the two-step problem solving. This means that we must first agree on the number of shares available per portfolio, and then determine the amount of investment in the shares in that portfolio. The chromosome designed in this paper solves the problem and solves the problem by creating the same structure for all chromosomes (Mehdi Khajezadeh Dezfuli, 2016). The following figure shows an example of a chromosome designed:



The designed structure consists of three main parts. In the first part of the chromosome, all existing companies in the problem (153 shares) are considered. The second part consists of a cell that is shown in green and is used to determine the number of shares in each portfolio, and the third part consists of orange cells that are used to select portfolios. All these cells are assigned a random number between 0 and 1, and then by the defined function, these values are converted to the main values of the problem. Initially, using the

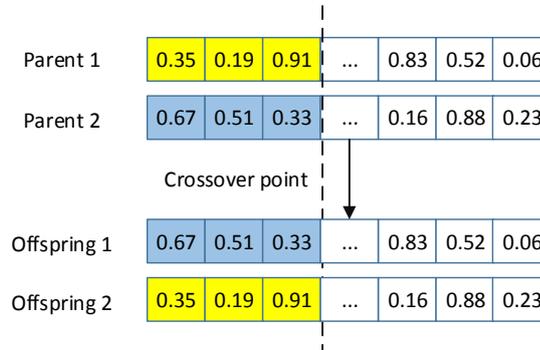
number contained in cell 154 and using the spacing taken between 0 and 1, and the location in each range, the number of shares in the portfolio is determined. Then, in the same way and with the function that is defined, the numbers in the orange cells determine the shares to be selected from among the existing 153 shares. At this point, a specific portfolio has been created, with numbers between 0 and 1; now, to determine the amount of investment per share, each of these numbers is divided by the total number of available portfolios in order to determine the percentage of investment per share. The total percentage values will be equal to one. Finally, using the values of the variables and the problem data, the objective functions of the model are calculated. The obvious characteristics of the chromosome can be pointed out that all the problem constraints in the created structure are observed, and that no irrelevant answer will be generated. So you can be sure that the final answer will be questionable.

4.1. Multi-objective genetic algorithm operators

Two operators perform the search process in the multi-objective genetic algorithm: mutation and crossover. The reproduction operator is used to extract better answers; while mutation operator is used to explore the broader response space, the crossover is used to extract better answers (Mehdi Khajezadeh Dezfuli, 2016).

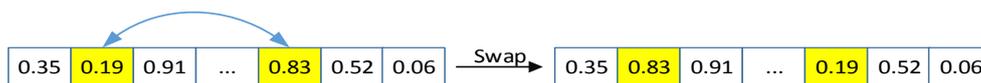
4.1.1. Crossover operator

Typically, this operator uses two chromosomes to create a chromosome with new features. Selected chromosomes are referred to as parents and a newly created chromosome is called the child. Several types of this operator are used in a multi-objective genetic algorithm which includes single point crossover, double point crossover and uniform crossover. In this paper, single point crossover operator is used to find better solutions. The parent roulette wheel selection is used to select parents. The following figure shows a sample of single-cut reproduction:



4.1.2. Mutation operator

There are several types of mutant operators that are used in the multi-objective genetic algorithm. Items such as Swap, Insertion and Inversion are used. All of these methods are used in this article. Firstly, a chromosome is selected using the roulette wheel method. The numbers 1 to 3 are assigned to mutant operators, respectively, and then a random number is generated between the interval (Enriqueta Vercher, Bermudez, 2015), which determines which operator to use. An example of a succession operator is shown in the following figure:



5. Research hypotheses

There is no significant difference between the performance of three-objective evolutionary models, mean-downside risk-skewness, mean-downside risk-kurtosis, fuzzy mean-fuzzy downside risk-fuzzy skewness and fuzzy mean-fuzzy downside risk- fuzzy kurtosis.

6. Sampling method

The population of the research is the industries listed in Tehran Stock Exchange with two criteria: A) Market concentration and turnover B) Number of companies accepted in the industry. Thus, the 35 industries listed on Tehran Stock Exchange, out with the exception of industries such as banks and credit institutions, insurance and pension funds, investment companies, Holdings and other financial intermediation (which are characterized by unusual capital structure and different reporting practices and can lead to data deviations), industries that are more exposed to market players and include at least 10 active and qualified companies were selected as the statistical sample of the survey.

7. Research method

This research is practical in terms of the purpose. In terms of the method of implementation, it is a descriptive research and it is a correlation analysis. This research can be considered as a post- traumatic research. The research steps were as follow:

1. In the first step, the data were collected.
2. Daily returns of each company were extracted from the TSETMC website and “Rahavard Novin” software, and the monthly return, downside risk, skewness and kurtosis for each company were calculated.
3. Quartiles of returns were determined and fuzzy return, fuzzy downside risk, fuzzy skewness, and fuzzy kurtosis were calculated and then, the numbers were converted to crisp numbers.
4. Interest of the Investor involved for investment in each company (up to 60% of the total initial arrival) and the number of portfolio members (between four and twelve shares).
5. The NSGA-II algorithm was coded in the MATLAB software. After Taguchi experiment design, the population size was 150, replication was 100, the reproductive rate was 85%, and the mutation rate was

15%. After that, each of the models was executed ten times and Pareto frontier was obtained.

6. The best portfolios were chosen from the Pareto frontier using the Sortino ratio.
7. To compare the extracted portfolios, the trainer's ratio was used, using the actual data of 2016-17, the summary of which is given in Table 1.
8. The variance analysis was used to compare the performance of the models.

8. Numeral results

In this section, all different scenarios examined will be compared in three-objective programming models and the results will be presented.

1. Results from three-objective planning models

Fuzzy Mean- Fuzzy downside risk- Fuzzy kurtosis	Fuzzy Mean - Fuzzy downside risk - Fuzzy skewness	Mean- downside risk- kurtosis	Mean-downside risk- skewness
22/73	33/41	5/54	19/65
Treyner ratio Fuzzy Mean- Fuzzy downside risk- Fuzzy kurtosis	Treyner retio of Fuzzy Mean - Fuzzy downside risk - Fuzzy skewness	Treyner ratio of Mean- downside risk- kurtosis	Treyner ratio of Mean-downside risk- skewness
16/58	-0/23	1/39	-6/57

Then the results of two different models of Mean-Downside risk-Skewness and Mean-Downside risk-Kurtosis will be compared:

2. T student test : Mean-Downsiderisk-Skewness and Mean-Downsiderisk-Kurtosis

ITEM	COMPARATIVE ITEMS	STATISTICAL VALUE	CRITICAL VALUE	Pvalue	APPROVED HYPOTHESES
1	Return of the models	3.81	2.262	0.0042	H1
2	Treyner's retio of the models	-4,20	2,26	0,0023	H1

As indicated in Table 2, the average returns from ten times implementation of the planning model of Mean-Downside risk-Skewness is much better than implementation of the planning model of Mean-Downside risk-Kurtosis, and create more return. The rejection of the assumption of H0 also shows that the results are not coincidental and the t test also confirms these results.

3. T student test : Fuzzy Mean-Fuzzy Downside risk- Fuzzy Skewness and Fuzzy Mean- Fuzzy Downside risk- Fuzzy Kurtosis

ITEM	COMPARATIVE ITEMS	STATISTICAL VALUE	CRITICAL VALUE	Pvalue	APPROVED HYPOTHESES
1	Return of the models	-2.6	2.26	0.029	H1
2	Treyner's retio of the models	2.47	2.26	0.036	H1

Table 3 shows that the H0 assumption is rejected in relation to the comparison of the two above models, which indicates that the difference between the obtained numbers is not accidental and reliable. Therefore, it can be concluded that the efficiency of the planning model is Fuzzy Mean-Fuzzy Downside risk-fuzzy Kurtosis is better than the Mean- Downside risk-Kurtosis.

4. T student test : Fuzzy Mean- Fuzzy Downside risk- Fuzzy Skewness and Mean- Downside risk -Skewness

ITEM	COMPARATIVE ITEMS	STATISTICAL VALUE	CRITICAL VALUE	Pvalue	APPROVED HYPOTHESES
1	Return of the models	-2.26	2.262	0.0497	H1
2	Treyner's retio of the models	6.37	2.26	0.00013	H1

Table 4 shows that the H0 assumption is rejected in relation to the comparison of the two above models, which indicates that the difference between the obtained numbers is not accidental and reliable. Therefore, it can be concluded that the efficiency of the planning model in Fuzzy average -

Fuzzy Downside risk - Fuzzy Skewness is better than the programming model Mean – Downside risk – Skewness.

5. T student test : Fuzzy Mean-Fuzzy Downside risk-Fuzzy Kurtosis and Mean-Downside risk-

Kurtosis

ITEM	COMPARATIVE ITEMS	STATISTICAL VALUE	CRITICAL VALUE	Pvalue	APPROVED HYPOTHESES
1	Return of the models	-3.45	2.26	0.007	H1
2	Treyner's ratio of the models	2,57	2.26	0,045935512	H1

As shown in Table 5, the average returns from ten times implementation of the planning model of fuzzy Mean-fuzzy downside risk-fuzzy Kurtosis, is better than Mean-Downside risk-Kurtosis.

9. Conclusion

Two important elements of investment are risk and returns. Investors are always willing to raise their returns at a certain level of risk, or at a certain level of return, reduce their risk. By presenting modern portfolio theory, Markowitz showed that creating a portfolio of financial assets would be possible to reduce a certain level of risk returns. Therefore, investors tend to maximize their expected returns and minimize risk by creating the optimal combination of financial assets in their stock portfolios. However, investors face three major problems: The first problem is the underlying assumption of the Markowitz model. That is, the return on assets is considered as a random number, which is far from reality in today's world due to uncertainty and ambiguity in existing information and future conditions. The second problem is the symmetry of the distribution function of assets and the investor's indifference to the risk criterion. The third problem is the solving methods used when considering and paying attention to the two previous problems. Considering the three problems raised, the present article tried to solve the

problems by using factors such as downside risk, skewness and kurtosis, and taking the considerations of the investor into account, and using fuzzy theory simultaneously to create optimal portfolios with higher return and lower risk. Finally, the obtained portfolios were compared in terms of performance with the Treynor's ratio, and it was determined that when we use fuzzy logic in the formation of portfolios, results were much better and more desirable and had more return and less risk.

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