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Investigating the Importance of Different Companies of Tehran Stock Exchange using Lower Tail Dependency based Interaction Network

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Abstract

Examining the importance and influence of financial market companies is one of the main issues in the field of financial management because sometimes the collapse of a stock exchange company can affect an entire financial market. One systematic way to analyze the significance and impacts of companies is to use complex networks based on Interaction Graphs (IGs). There are different methods for quantifying the edge weight in an IG. In this method, the graph vertices represent the stock exchange companies that are connected by weighted edges (corresponding to the extent to which they relate to each other). In this paper, using the GARCH model (1,1) and the Clayton copula, we obtained the lower tail dependence interaction network of the first 52 companies of the Tehran Stock Exchange in terms of average market value, between June 2017 and October 2020. Then, based on the minimum spanning tree of the interaction network, we divided the companies into different communities. Using this classification, it was observed that the companies of the first group (Food Industry) and the second group (Oil Refinery) have the greatest impact on other companies. We also calculated the central indexes of the minimum spanning tree for each company. According to the results, the companies of the third group (Steel) have the highest average in the central indicators.

Keywords: Interaction network, Minimum Spanning Tree, GARCH Model, Clayton Copula, Lower Tail Dependence.

Introduction

Examining the importance and influence of financial market companies is one of the main issues in the field of financial management. Because sometimes the collapse of a stock exchange company can affect an entire financial market. An example is the collapse of Lehman Brothers Holding, which eventually led to the 2008 global financial crisis. One systematic way to analyze the significance and impacts of companies is to use complex networks based on Interaction Graphs (IGs) (Yang et al., 2020). In this method, the graph vertices represent the stock exchange companies that are connected by weighted edges (corresponding to the extent to which they relate to each other). Converting an IG to a tree in which loops have been removed reduces the complexity of the problem and simplifies system analysis. In this study, we used the Minimum Spanning Tree (MST) to analyze the results.

There are different methods for quantifying the edge weight in an IG. Among these methods, we can mention the Pearson Correlation (PerC) coefficient (Patro et al., 2013; Wiliński et al., 2013) and, Transfer Entropy (TE) (Ardalankia et al., 2020; Kwon & Yang, 2008; Osoolian & Koushki, 2020). Here we use Lower Tail Dependence (LTD) to measure the edge weight in the IG, which is a measure of the coordinated behavior of companies in negative returns (Wang & Xie, 2016). This quantity can be used in portfolio risk management. To calculate LTD, we need models through which we can obtain the marginal and joint Probability Distribution Functions (PDFs) of the returns of companies. Due to the fat-tailed behavior of the return PDF in financial markets (Chakraborti et al., 2011), we have used the GARCH(1,1) model to calculate the marginal PDFs (Lee & Hansen, 1994). To obtain joint PFDs we used the Clayton copula (Wen et al., 2019).

The structure of the present article is as follows: First, we briefly review using IGs in financial market analysis. Next, we have introduced the quantities and models used. Then, the results of the research are presented. And finally, we discuss and conclude the results.

Literature Review

The idea of using IGs in financial market analysis goes back to Mantegna. Using PerC-based IG, he was able to obtain a hierarchical structure for the companies under study. In other words, he was able to use the IG to divide companies into groups that had the same field of activity (Mantegna, 1999).

Bonanno et al. investigated the topological structure of the MST in financial markets. They have shown that for real markets, MST has a structure that cannot be reconstructed from random data, even in the first approximation (Bonanno et al., 2003).

Wiliński et al. have shown that the MST of the Frankfurt Stock Exchange has a unique topological structure (superstar) during the financial crisis between 2007 and 2008 (Wiliński et al., 2013). Therefore, it can be used to predict the occurrence of financial crises. A similar study on the impact of the global financial crisis on the South Korean financial markets IG has been conducted by Nobi et al. (Nobi et al., 2014).

Kwon and Yang have used the TE-based IG to determine the direction of information flow in financial markets. They have shown that information flows from US financial markets to Asian financial markets. They also use the MST to show that the S&P500 market is the primary source of information among financial markets (Kwon & Yang, 2008).

Wang et al. investigated the differences between the PerC-based and Partial Correlation (ParC)-based MST. Based on central indexes, they have shown that the ParC-based MST provides more reliable results than the PerC-based MST (Wang et al., 2018).

Eng-Uthaiwat has shown that the topology of the IG can be used to predict the return of the portfolio (Eng-Uthaiwat, 2018). Peralta and Zareei have also presented a method for the optimal selection of the portfolio using the IG (Peralta & Zareei, 2016).

Research Methodology

The statistical population of this study is the 100 large companies of the Tehran Stock Exchange based on the average market value (https://www.fipiran.ir/). Among them, companies that had more than 3 months of interruption in daily stock exchange transactions were eliminated. Finally, the remaining 52 companies were selected as statistical sample companies. For these companies, the LTD-based IG was drawn. We used the Clayton copula and the GARCH(1,1) model to calculate the LTD. To obtain the parameters of the Clayton copula and the GARCH(1,1) model, we used the Maximum Likelihood Estimation (MLE) method. From the IG, we obtained the MST for simplicity in analyzing the results. Using MST, we divided the studied companies into different groups in terms of dependency. Finally, we calculated the central indexes associated with the MST.

1. Interaction Network Construction

- **Lower Tail Dependence**

In probability theory, the LTD of a pair of random variables is a measure of their coordinated motion in the tail of the probability distribution. The LTD (λ_l) of a pair of random variables is defined as follows:

$$\lambda_l := \lim_{q\to 0} \Pr[Y \le F_Y^{\leftarrow}(q) | X \le F_X^{\leftarrow}(q)], \qquad (1)$$

where $F_X^{\leftarrow}(q) = \inf\{x \in \mathbb{R}: F_X(x) \ge q\}$ is the inverse of the Cumulative Distribution Function (CDF) $F_X(x)$.

Similarly, the Upper Tail Dependence (UTL) λ_u is defined as follows:

$$\lambda_u = \lim_{q\to 1} \Pr[Y > F_Y^{\leftarrow}(q) | X > F_X^{\leftarrow}(q)]. \qquad (2)$$

- **Copula**

The joint CDF of several continuously uniform random variables on the set [0,1] is called a Copula. In other words, if $(U_1, U_2, \dots, U_d)$ is a random vector with $U_i \sim U[0,1]$, then the copula

$C: [0,1]^d \to [0,1]$ is defined as follows:

$$C(u_1, u_2, \dots, u_d) := \Pr[U_1 \le u_1, U_2 \le u_2, \dots, U_d \le u_d]. \qquad (3)$$

See Appendix 1 for details on Copula.

- **GARCH(p,q) Model**

In economics, the Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH) is a statistical model for time series data. In this model, the time series of the return of a financial institution X_t is given as follows:

$$X_t = e_t \, \sigma_t, \tag{4}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2, \tag{5}$$

where $e_t \sim N(0,1)$ (random variable with normal distribution with mean zero and variance one), σ_t^2 is the time-dependent variance, and $\{\omega, \alpha_i, \beta_j\}$ are the free parameters of the model. These parameters can be estimated using the real value of X_t in a time series based on the MLE method.

As can be seen, in this model the variance of the random variable X_t, itself is also a random variable. It can be shown that in the GARCH(1,1) model the mean value of variance is equal to $\frac{\omega}{1-\alpha-\beta}$. In obtaining the time series of variance, we use this value as the starting point of variance. In other words, we put $\sigma_1^2 = \frac{\omega}{1-\alpha-\beta}$.

- **Maximum Likelihood Estimation**

Suppose X is a random variable with PDF $P_X(x|\theta)$ that depends on the parameter θ. If $\{x_1, x_2, \dots, x_n\}$ are the results of n experiments on the random variable X, we want to get the best estimate for the value of the parameter θ in terms of these results. In the Maximum Likelihood Estimation method, the best estimate for the parameter θ is the value that maximizes the following likelihood function:

$$\begin{aligned} L(\theta|x_1, x_2, \dots, x_n) &\coloneqq \log[\Pr(x_1, x_2, \dots, x_n|\theta)] \\ &= \log[P_X(x_1|\theta) P_X(x_2|\theta) \dots P_X(x_n|\theta)] \\ &= \sum_{i=1}^{n} \underbrace{\log[P_X(x_i|\theta)]}_{=l(\theta|x_i)}. \end{aligned} \tag{6}$$

2. Assortativity

For a graph whose vertices are divided into groups, the assortativity coefficient is a measure that quantifies the quality of the grouping. The value of this coefficient is in the set [0,1]. The closer the value to one indicates more edges in groups relative to edges that connect the groups. The assortativity coefficient of a connected graph is defined as follows:

$$r = \frac{\sum_{i,j}\left(a_{i,j} - \frac{d_i d_j}{2m}\right) f_{i,j}}{\sum_{i,j}\left(d_i \delta_{i,j} - \frac{d_i d_j}{2m}\right) f_{i,j}}, \tag{7}$$

where m is the total number of edges, $a_{i,j}$ is the weight of the edge connecting vertices v_i and v_j, d_i is the degree of the vertice v_i, $\delta_{i,j}$ is the Kronecker delta, and, $f_{i,j}$ is a binary function that is equal to one when vertices v_i and v_j belong to the same group otherwise, it is equal to zero.

3. Central Indexes

- **Node Degree**

The degree of each vertice or node is equal to the number of edges connected to that node. The degree of node v_i is defined as follows:

$$d(v_i) = \sum_j \delta_{i,j}. \tag{8}$$

- **Node Strength**

The strength of node v_i is defined as follows:

$$s(v_i) = \sum_j \delta_{i,j} (\lambda_l)_{i,j}, \tag{9}$$

where $(\lambda_l)_{i,j}$is the LTD of nodes v_i and v_j.

- **Betweenness Centrality**

Betweenness centrality is the way to determine the extent to which a node affects the flow of information in a graph. It is often used to find groups that act as bridges from one part of a graph to another. The betweenness centrality of node v_i is defined as follows:

$$c_B(v_i) = \sum_{s,t} \frac{n_{s,t}^{(i)}}{n_{s,t}}, \tag{10}$$

where $n_{s,t}$ is the number of the shortest path from node v_s to the node v_t and, $n_{s,t}^{(i)}$ is the number of the shortest path from node v_s to the node v_t that crosses the node v_i.

- **Closeness Centrality**

Closeness centrality shows the average distance of that vertex from other vertices. The larger the closeness centrality of a node, the closer that node is to the other nodes. The closeness centrality of node v_i is defined as follows:

$$c_M(v_i) = \frac{k(v_i)}{\sum_j l_{i,j}}, \tag{11}$$

where $l_{i,j}$ is the distance of the shortest path from node v_i to node v_j.

Data Analyses

1. Interaction Network Construction

The data used in this study is the time series of the closed stock prices of 52 companies on the Tehran Stock Exchange in the period 2017/6/22 to 2020/10/21. Using this data, we obtained the logarithmic time series of stock prices of companies as follows:

$$X_t = \log S_i - \log S_{i-1}. \tag{12}$$

Then, using the GARCH(1,1) model (Eq.9 and Eq.10), we calculated the time series of variance through the following recursive relation:

$$\sigma_t^2 = \omega + \alpha\, X_{t-1}^2 + \beta\, \sigma_{t-1}^2, \quad t \geq 2 \tag{13}$$

$$\sigma_1 = \frac{\omega}{1-\alpha-\beta}. \tag{14}$$

In this model, X_t is a random variable with a normal distribution function with a mean of zero and a variance of σ_t^2, in other words

$$P(x_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{x_t^2}{2\sigma_t^2}\right). \tag{15}$$

We used the MLE method to estimate the parameters $\theta = \{\omega, \alpha, \beta\}$. The best estimates for the value of these parameters using the X_t time series are values that maximize the following likelihood function:

$$L(\theta|x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} l(\theta|x_i),$$

$$l(\theta|x_i) = \log P(x_i) = -\frac{1}{2}\log(2\pi\sigma_i^2) - \frac{x_t^2}{2\sigma_t^2}. \tag{16}$$

The values obtained for these parameters for each company are given in See Appendix 2 for company names. **Error! Reference source not found.**

In this study, we used the Clayton copula to calculate the LTD of two companies. We used the MLE method to obtain the θ parameter of the Clayton copula between the two companies. The likelihood function used to estimate the parameter θ is as follows:

$$L(\theta|x_1, y_1, x_2, y_2, \dots, x_n, y_n) = \sum_{i=1}^{n} l(\theta|x_i, y_i),$$

$$l(\theta|x_i, y_i) = \log c(u, v) = \log\left[(1+\theta)(uv)^{-1-\theta}\left(u^{-\theta} + v^{-\theta} - 1\right)^{-2-\frac{1}{\theta}}\right], \tag{17}$$

where u and v are the normal CDF for the return of companies X and Y, respectively, and are obtained as follows:

$$u(x_i) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{x_i}{\sqrt{2\,\sigma_i^2(X)}}\right)\right), \quad v(y_i) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{y_i}{\sqrt{2\,\sigma_i^2(Y)}}\right)\right),$$

where erf is the error function.

After obtaining the parameter θ for each pair of companies, we obtained their LTD using Eq.8. The mean, variance, maximum, and, minimum of LTDs are given in Table 1.

Table 1. LTD statistics of companies

Mean	Variance	Min	Max
0.199	0.164	0.000	0.878

To draw the MST, we first converted the LTD matrix to the distance matrix as follows:

$$d_{i,j} = \sqrt{2\left(1 - (\lambda_l)_{i,j}\right)}. \quad (18)$$

Figure 1 shows this matrix. The corresponding MST is depicted in Figure 2.

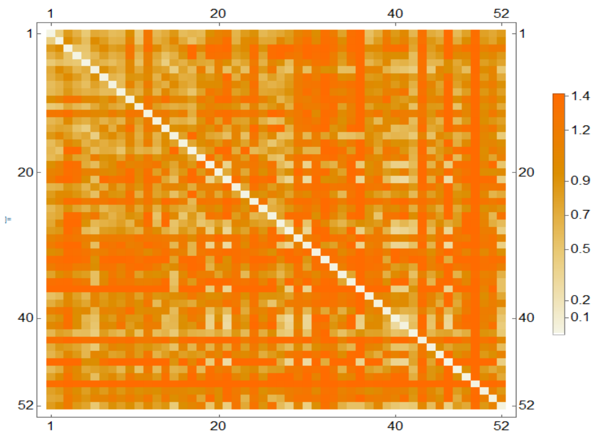

Figure 1. LTD-Distance Matrix

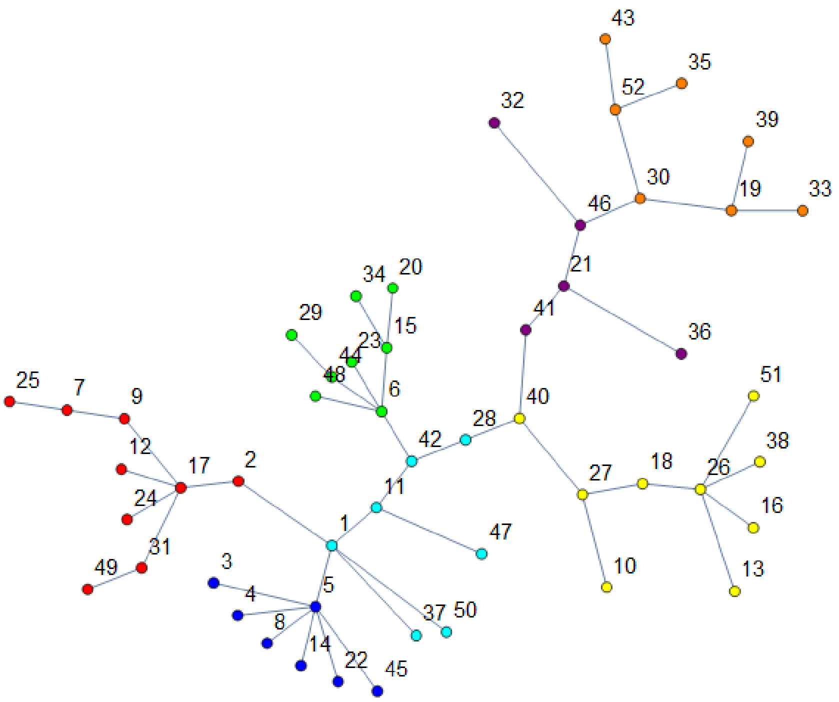

Figure 2. LDT-Based MST

Using the FindGraphCommunities command in Mathematica software, the present companies were divided into different groups on MST. These groups are listed in Table 2 along with the mean value of the LTD of each group.

Table 2. Dividing companies into different groups on MST

Group Number	Group Name	Companies	Mean LTD
1	Food Industry	Glucosan (21), Pars Minoo Industrial Co. (32), Iran China Clay Ind. (36), Iran Khodro Investment Development Co. (41), Shahid Ghandi Production Factories Co. (46)	0.556
2	Oil Refinery	Isfahan Oil Refinery (3), Tehran Oil Refinery (4), Bandar Abbas Oil Refinery (5), Parsian Oil and Gas Development Group (8), Tabriz Oil Refinery (14), Civil Pension Fund Investment (22), Khark Petrochemical Co. (45)	0.389

3	Steel	Mobarakeh Steel Co. (1), Khouzestan Steel Co. (11), Behran Oil Co. (28), Iran Alloy Steel Co. (37), Sepahan Oil Co. (42), South Kaveh Steel Co. (47), Iranian Investment Petrochemical Group (50)	٠.٣٧٩
4	Communication	Mobile Telecommunication Company of Iran (6), Informatics Services Co. (15), Golrang Industrial Group (20), Iran Chemical Industries Investment Co. (23), National Investment Co. (29), Asan Pardakht Persian (34), Telecommunication Company of Iran (44), MAPNA Group (48)	٠.٣١٥
5	Petrochemical	Omid Investment Co. (10), Jam company (13), Pardis Petrochemical Co. (16), Mobin Petrochemical Co. (18), Shazand Petrochemical Co. (26), Middle East Bank (27), Kermanshah Petrochemical Ind. (38), Iran Khodro (40), Persian Gulf Fajr Energy Co. (51)	٠.٣٠٧
6	Pharmacy	Dr. Abidi Pharmacy (19), Carton Iran (30), Pars Oil (33), Bahman Group (35), Zahravi Pharmaceutical Co. (39), Tehran Stock Exchange (43), Iranian Aluminium Co. (52)	٠.٢٦٤
7	Investment	National Iranian Copper Industries Co. (2), Tamin Petroleum & Petrochemical Investment Co. (7), Ghadir Investment Co. (9), Islamic Republic of Iran Shipping Line Group (12), National Development Investment Group (17), Tamin Pharmaceutical Investment Co. (24), Shiraz Petrochemical Co. (25), Iran Transfo (31), Mobarakeh Steel Co. (49)	٠.٢٠٠

Considering that only the food companies in the list of 52 companies i.e. Glucosan and Pars Minoo Industrial Co. were in the first group, we named this group Food Industry. A similar argument is made about the Pharmacy group. The nomination of other groups is selected according to the number of dominant companies in that group that have the same field of activity.

2. Assortativity

The assortativity coefficient of grouping was 0.897. This value indicates that the grouping performed has a high-resolution quality.

3. Central Indexes

Table 3 shows the average of the central indexes for the classified groups in

Table 2. The highest node strength is related to the first group (Food Industry) and the highest node degree, betweenness centrality, and, closeness centrality is related to the third group (Steel). Table 4 also shows the top five companies in each central index. As can be seen, Shazand Petrochemical Co. is among the top 5 companies in three of the four central indexes. Bandar Abbas Oil Refinery and Iran Khodro Investment Development Co. are also among the top five companies in two of the four central indexes.

Table 3. Average central indexes of groups

Group	**Node Degree**	**Node Strength**	**Betweenness Centrality**	**Closeness Centrality**
1	۲,۰۰۰	۱,۵۲۱	۲۴۲	۰,۲۱۷
2	۱,۸۵۷	۱,۰۵۰	۴۱	۰,۱۹۱
3	۲,۲۸۶	۱,۳۵۳	۳۹۵	۰,۲۴۹
4	۱,۸۷۵	۰,۹۷۸	۵۹	۰,۲۰۰
5	۲,۰۰۰	۱,۱۶۹	۱۶۱	۰,۱۸۵
6	۱,۸۵۷	۱,۰۲۹	۶۸	۰,۱۶۵
7	۱,۸۸۹	۰,۸۵۱	۹۶	۰,۱۶۷

Table 4. The first 5 companies in the central indexes

Rank	**Node Degree**	**Node Strength**	**Betweenness Centrality**	**Closeness Centrality**
1	Bandar Abbas Oil Refinery	Shazand Petrochemical Co.	Iran Khodro Investment Development Co.	South Kaveh Steel Co.
2	Mobarakeh Steel Co.	Khouzestan Steel Co.	Iranian Investment Petrochemical Group	Mobin Petrochemical Co.
3	Mobile Telecommunication Company of Iran	Bandar Abbas Oil Refinery	Shazand Petrochemical Co.	Persian Gulf Fajr Energy Co.
4	National Development Investment Group	Parsian Oil and Gas Development Group	Carton Iran	Telecommunication Company of Iran
5	Shazand Petrochemical Co.	Iran Transfo	Pars Minoo Industrial Co.	Iran Khodro Investment Development Co.

Discussion and Conclusion

In the study, the impact of the selected 52 companies from the Tehran Stock Exchange (based on market value) on each other in the period from June 2017 to October 2020 was examined. The mathematical tool used in this study was the lower tail dependence of companies. This quantity was calculated using the GARCH(1,1) model and the Clayton copula. By obtaining the lower tail dependence of the companies, their interaction graph was obtained. By drawing the minimum spanning tree from the interaction graph, the existing companies were divided into different groups based on their impacts on each other. The assortativity coefficient of the minimum spanning tree in this division was 0.897, which indicates the appropriate resolution of the groups. According to the results, it can be seen that the companies in the first (Food Industry) and second (Oil Refinery) groups have the greatest impact on other companies in the period under review. Also, companies in the sixth (Pharmacy) and seventh (Investment) groups have the least impact on other groups. By calculating the central indexes, it was observed that the third group (Steel) had the highest average in three indexes (node degree, betweenness centrality, and, closeness centrality) of the four studied indexes. Also, Shazand Petrochemical Co., Bandar Abbas Oil Refinery, and, Iran Khodro Investment Development Co. were among the first companies in the central indexes.

Based on the above results, the policy implications of this article are as follows:

a. By systematically categorizing companies based on lower tail dependencies, regulators can apply different protection policies to different groups in times of financial crisis.

b. By identifying key nodes with respect to central indexes, financial regulators can effectively monitor to reduce the spread

Declaration of Conflicting Interests

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Appendix 1: Copula

The copula has the following properties:

- The copula $C(u_1, u_2, \dots, u_d)$ is a non-descending function of any variable u_i.
- If all variables of the copula are equal to one, except one variable, then the value of the copula is equal to that variable. In other words, $C(1,1,\dots,1,u_i,1,\dots,1) = u_i$.
- For each $a_i \leq b_j$ the copula C holds in the following rectangular inequality:

$$\sum_{i_1=1}^{2} \sum_{i_2=1}^{2} \dots \sum_{i_d=1}^{2} (-1)^{i_1+i_2+\dots+i_d} C\left(u_{1,i_1}, u_{2,i_2}, \dots, u_{d,i_d}\right) \geq 0,$$

where $u_{j,1} = a_j$ and $u_{j,2} = b_j$.

The third property arises from the positivity of the probability $\Pr\left[u_1 \in [a_1, b_1], u_2 \in [a_2, b_2], \dots, u_d \in [a_d, b_d]\right]$. To better understand this feature, the following two variables example can be useful.

$$\begin{aligned}
&\Pr\left[u_1 \in [a_1, b_1], u_2 \in [a_2, b_2]\right] \\
&\quad = \Pr\left[u_1 \leq b_1, u_2 \in [a_2, b_2]\right] - \Pr\left[u_1 \leq a_1, u_2 \in [a_2, b_2]\right] \\
&\quad = (\Pr[u_1 \leq b_1, u_2 \leq b_2] - \Pr[u_1 \leq b_1, u_2 \leq a_2]) \\
&\quad - (\Pr[u_1 \leq a_1, u_2 \leq b_2] - \Pr[u_1 \leq a_1, u_2 \leq a_2]) \\
&\quad = (-1)^{1+1} \Pr[u_1 \leq a_1, u_2 \leq a_2] \\
&\quad + (-1)^{2+1} \Pr[u_1 \leq b_1, u_2 \leq a_2] \\
&\quad + (-1)^{1+2} \Pr[u_1 \leq a_1, u_2 \leq b_2] \\
&\quad + (-1)^{2+2} \Pr[u_1 \leq b_1, u_2 \leq b_2] \\
&\quad = (-1)^{1+1} C(a_1, a_2) + (-1)^{2+1} C(b_1, a_2) + (-1)^{1+2} C(a_1, b_2) \\
&\quad + (-1)^{2+2} C(b_1, b_2) \geq 0.
\end{aligned}$$

The reverse is also true. In other words, any function $C: [0,1]^d \rightarrow [0,1]$ that holds in the above three properties is a copula. It can also be easily shown that if $C(u_1, u_2, \dots, u_d)$ is a d-dimensional copula, then $C(1, u_2, \dots, u_d)$ is a $(d-1)$-dimensional copula.

In the following, we state the important Sklar theorem on copulas by proving two simple lemmas.

Lemma 1: If $X \sim U[0,1]$ then $F_X(x) = x$.

Proof: $F_X(x) = \Pr[X \leq x] = \int_0^x P_X(x)dx = \int_0^x 1\, dx = x$.

Lemma 2: If $Y = F_X(X)$ then $Y \sim U[0,1]$.

Proof:$F_Y(y) = \Pr[Y \leq y] = \Pr[F_X(X) \leq y] = \Pr[X \leq F_X^{-1}(y)] = F_X\left(F_X^{-1}(y)\right) = y$.

Sklar Theorem: If $\mathrm{X} = \{\mathrm{X}_1, \mathrm{X}_2, \dots, \mathrm{X}_\mathrm{d}\}$ is the set of d random variables with CDFs given by F_i, then there exists a copula C, such that:

$$\mathrm{F}(\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_\mathrm{d}) = \mathrm{C}\left(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_2(\mathrm{x}_2), \dots, \mathrm{F}_\mathrm{d}(\mathrm{x}_\mathrm{d})\right), \tag{19}$$

where F is the joint CDF of the set X:

$$\mathrm{F}(\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_\mathrm{d}) \coloneqq \Pr[\mathrm{X}_1 \leq \mathrm{x}_1, \mathrm{X}_2 \leq \mathrm{x}_2, \dots, \mathrm{X}_\mathrm{d} \leq \mathrm{x}_\mathrm{d}]. \tag{20}$$

As mentioned, the copula is a function for calculating the joint CDF of several random variables. Similarly, copula density is a function for calculating the joint PDF of several random variables. If $\mathrm{C}(\mathrm{u}, \mathrm{v})$ is the copula of two random variables, then the copula density of these two random variables $\mathrm{c}(\mathrm{u}, \mathrm{v})$ is defined as follows:

$$\mathrm{c}(\mathrm{u}, \mathrm{v}) = \frac{\partial^2 \mathrm{C}(\mathrm{u}, \mathrm{v})}{\partial \mathrm{u}\ \partial \mathrm{v}}. \tag{21}$$

The joint PDFof the two random variables X and Y is related to their copula density as follows:

$$\mathrm{P}_{\mathrm{XY}}(\mathrm{x}, \mathrm{y}) = \mathrm{c}(\mathrm{u}, \mathrm{v})\ \mathrm{P}_\mathrm{X}(\mathrm{x})\ \mathrm{P}_\mathrm{Y}(\mathrm{y}), \tag{22}$$

where $u = F_X(x)$ and $v = F_Y(x)$.

Proof: $P_{XY}(x, y) = \frac{\partial^2 F_{XY}(x,y)}{\partial x\, \partial y} = \frac{\partial^2 C(u,v)}{\partial u\, \partial y} \frac{\partial F_X(x)}{\partial x} \frac{\partial F_Y(y)}{\partial y} = c(u, v)\ P_X(x)\ P_Y(y)$.

Error! Reference source not found. lists some well-known two-variable copulas with their densities. The parameters used in these copulas can be obtained from the time series of random variables using the MLE method.

Table 6. Some two-variable copulas with their densities

Copula	$C(u,v)$	$c(u,v)$	Parameter
Normal	$=\Phi_2(\Phi^{-1}(u),\Phi^{-1}(v);\varrho)$, where $\Phi_2(h,k;\varrho)=\int_{-\infty}^{h}\int_{-\infty}^{k}\phi_2(x,y;\varrho)dy\,dx$, $\phi_2(x,y;\varrho)=\frac{1}{2\pi\sqrt{1-\varrho^2}}\exp\left(-\frac{x^2-2\varrho xy+y^2}{2(1-\varrho^2)}\right)$, $\Phi(h)=\int_{-\infty}^{h}\phi(x)\,dx$, $\phi(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$.	$=\frac{1}{\sqrt{1-\varrho^2}}\exp\left(-\frac{(a^2+b^2)\varrho-2ab\varrho}{2(1-\varrho^2)}\right)$ where $a=\sqrt{2}\,\mathrm{erf}^{-1}(2u-1)$, $b=\sqrt{2}\,\mathrm{erf}^{-1}(2v-1)$, $\mathrm{erf}(z)=\frac{2}{\sqrt{\pi}}\int_0^z\exp(-t^2)\;dt$.	$\varrho\in[-1,1]$
FGM[1]	$=uv[1+\theta(1-u)(1-v)]$.	$=1+\theta(2u-1)(2v-1)$.	$\theta\in[-1,1]$
Frank	$=-\frac{1}{\theta}\log\left(1+\frac{(\exp(-\theta u-1))\,(\exp(-\theta v-1))}{\exp(-\theta)-1}\right)$.	$=\frac{\theta\exp(\theta(u+v))(\exp(\theta)-1)}{(\exp(\theta)-\exp(\theta u)-\exp(\theta v)+\exp(\theta(u+v)))}$	$\theta\in\mathbb{R}\backslash\{0\}$
Clayton	$=\left(\max\{u^{-\theta}+v^{-\theta}-1;0\}\right)^{-\frac{1}{\theta}}$.	$=(1+\theta)(uv)^{-1-\theta}\left(u^{-\theta}+v^{-\theta}-1\right)^{-2-\frac{1}{\theta}}$.	$\theta\in(-1,\infty)\backslash\{0\}$

The LTD of two random variables with Clayton copula is as follows:

$$\lambda_l=2^{-\frac{1}{\theta}}. \quad (23)$$

Appendix 2: Companies' Names

[1] Farlie-Gumbel-Morgenstern

Number	Company
01	Mobarakeh Steel Co.
02	National Iranian Copper Industries Co.
03	Isfahan Oil Refinery
04	Tehran Oil Refinery
05	Bandar Abbas Oil Refinery
06	Mobile Telecommunication Company of Iran
07	Tamin Petroleum & Petrochemical Investment Co.
08	Parsian Oil and Gas Development Group
09	Ghadir Investment Co.
10	Omid Investment Co.
11	Khouzestan Steel Co.
12	Islamic Republic of Iran Shipping Line Group
13	Jam company
14	Tabriz Oil Refinery
15	Informatics Services Co.
16	Pardis Petrochemical Co.
17	National Development Investment Group
18	Mobin Petrochemical Co.
19	Dr. Abidi Pharmacy
20	Golrang Industrial Group
21	Glucosan
22	Civil Pension Fund Investment
23	Iran Chemical Industries Investment Co.
24	Tamin Pharmaceutical Investment Co.
25	Shiraz Petrochemical Co.
26	Shazand Petrochemical Co.
27	Middle East Bank
28	Behran Oil Co.
29	National Investment Co.
30	Carton Iran
31	Iran Transfo
32	Pars Minoo Industrial Co.
33	Pars Oil
34	Asan Pardakht Persian
35	Bahman Group
36	Iran China Clay Ind.
37	Iran Alloy Steel Co.
38	Kermanshah Petrochemical Ind.
39	Zahravi Pharmaceutical Co.
40	Iran Khodro
41	Iran Khodro Investment Development Co.
42	Sepahan Oil Co.
43	Tehran Stock Exchange
44	Telecommunication Company of Iran
45	Khark Petrochemical Co.

46	Shahid Ghandi Production Factories Co.
47	South Kaveh Steel Co.
48	MAPNA Group
49	Mobarakeh Steel Co.
50	Iranian Investment Petrochemical Group
51	Persian Gulf Fajr Energy Co.
52	Iranian Aluminium Co.